The Working Principle of Ant Colony Optimization

May 29, 2013

The Ant Colony Optimization Algorithm (ACO) is an probabilistic computational optimization technique to solve some path finding problem.

The ACO is one of the example of *Swarm Intelligent System*.

1 How do ants communicate : The pheromone

The real ant will secrete a kind of chemical called "pheromone" for communication.

The secreting ant will secrete pheromone.

The receving ant's behavior will be changed, it is more likely to follow the pheromone.

2 The Principle

Initially the ants are all concentrated in the colony, and they need to find the location of food and bring the food home.

In the begining, all ant move randomly.

As the ant moves, it will lay down pheromone trials.

Pheromone is attractive to ant : when other ants find move to the position with pheromone, they will tend to move along the path instead of moving in a random pattern.

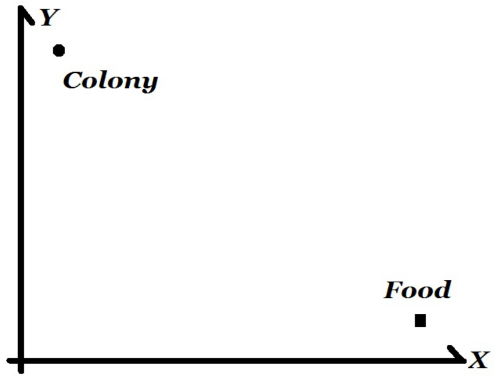
The pheromone will evaporate overtime, thus reducing the attractiveness to the ant.

After some time (some iterations), the path with better performance will be strengthen (since ants will follow the path more likely as well as lay down pheromone to strengthen the attraction, this is the positive feedback !) , the path with poor performance will disappear (less attractive to ant + pheromone evapouration)

And eventually, the ants will find the optimal path.

3 Detail explanations

Initially the ants are all concentrated in the colony, and they need to find the location of food and bring the food home, and the ant has no knowledge of the location of the food.

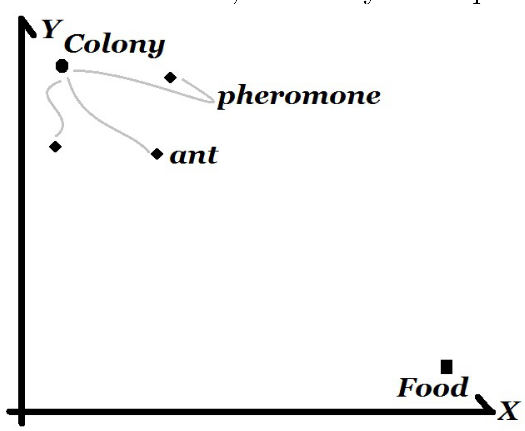


Then the ant start to move at random, this can be modeled as random walks

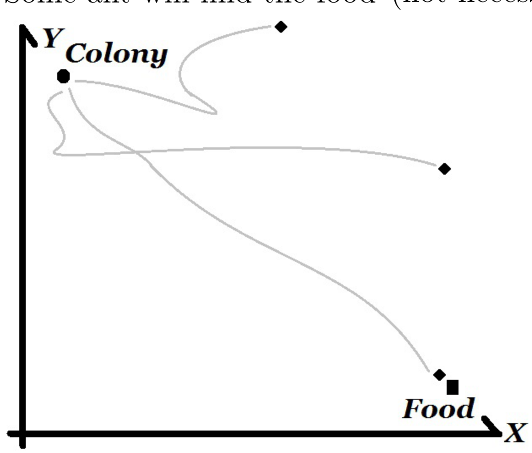
$$X_{k+1} = X_k + hD_k$$

(i.e. the next location $X_{k+1} = (x_{k+1}, y_{k+1})$ is the current location $X_k = (x_k, y_k)$ plus the random step which is the product of step size h and direction $D_k = (d_{x,k} + d_{y,k})$, where D_k is generated by random number generator)

As the ant move, it will lay down pheromone trials.

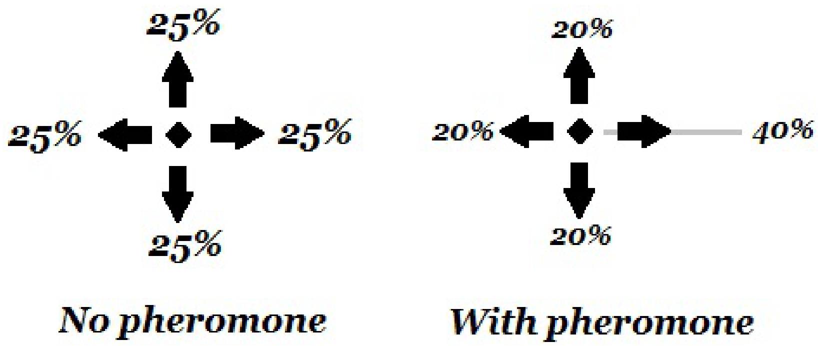


Some ant will find the food (not necessary the optimal solution)



Direction Selection

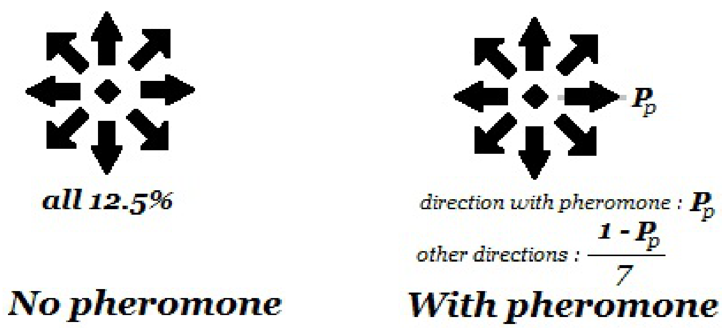
Now consider the ant moving method with and with out pheromone. There are lots of direction selection scheme , for example, consider the simple 4-direction selection scheme :



When there is no pheromone, the ant just perform random walk, thus the chance of moving to any direction is equal.

When there is pheromone, the chance of the ant to travel along the pheromone trials will be higher, for example, 40%. And other remaining directions will be lower.

The above method is simple and easy to implement, in general, the scheme can be implemented as

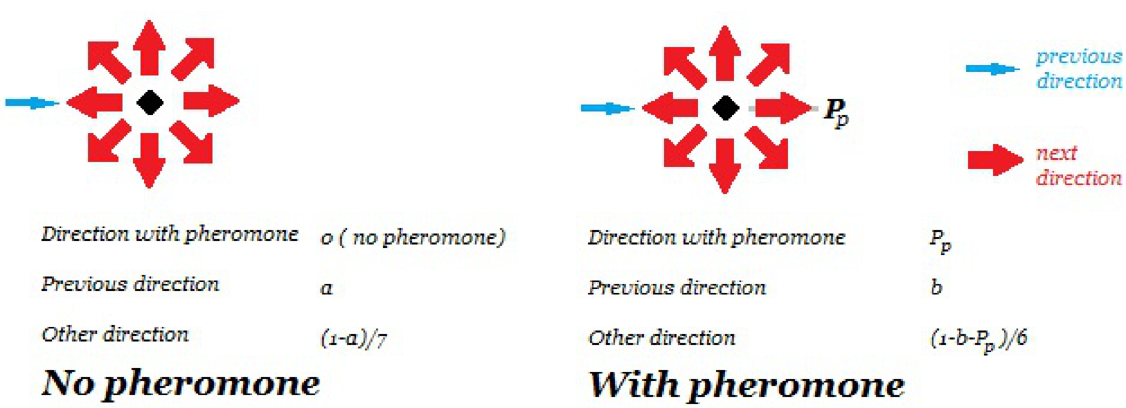


In the above scheme, there 8 possible directions (since in pixel geometry, there are 8 directions in total !)

The chance of picking any direction with no pheromone is evenly distributed

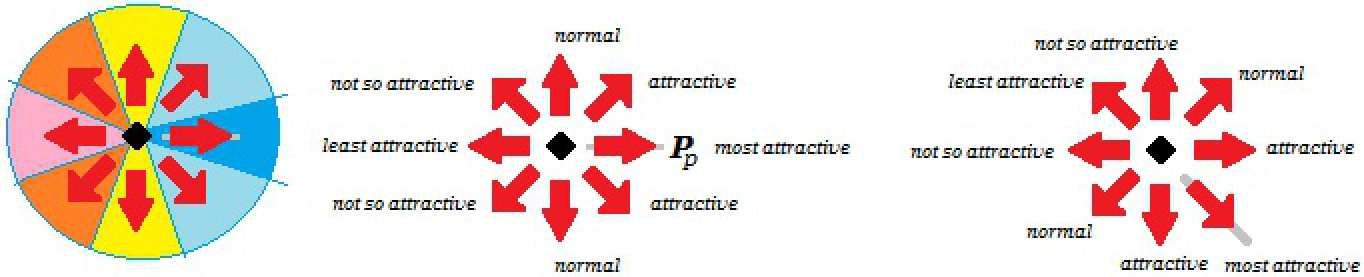
When there is pheromone, the chance of picking the direction with pheromone is higher : P_p which is the probability from the preference function ($P_p \in [0, 1]$), and all the remaining 7 direction has equal chance of being selected : $\frac{1 - P_p}{7}$

The above scheme is already good enough, but further improvement can be made by lowering the chance of the ant to "turn back"



Where a, b can be a constant or output of some preference function

The scheme can be made even more complicated if the pheromone also attract the ant to move toward the pheromone direction



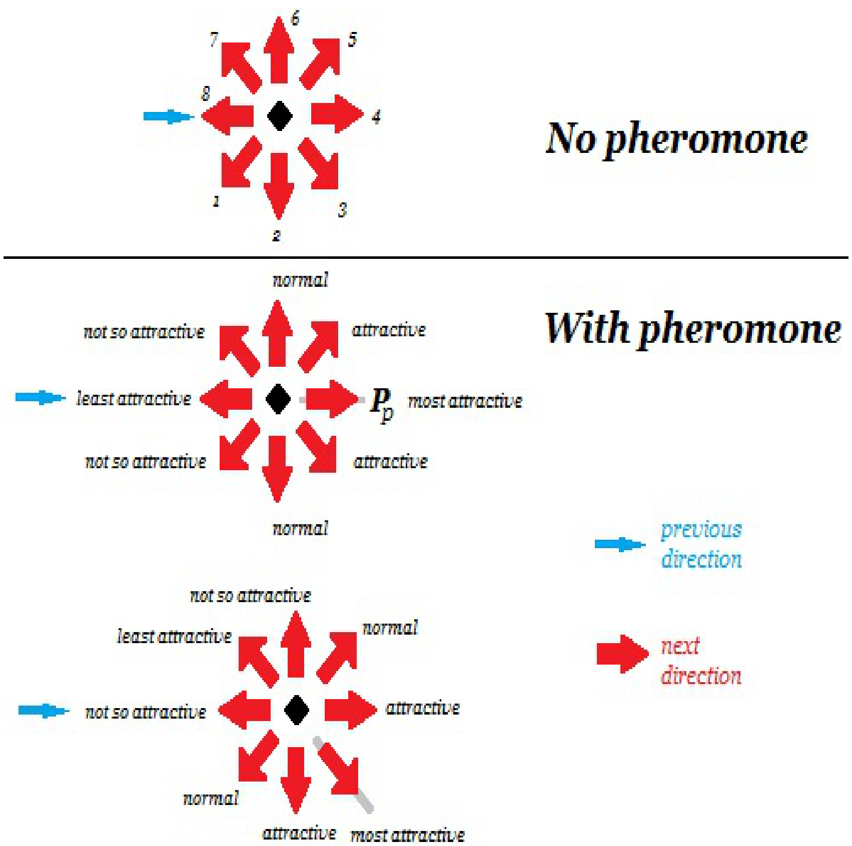
Where the chance of the ant to move in specific direction is determined by the preference function.

The preference function can be deterministic, or fuzzy. For example, the preference function can be an gaussian or discrete.

$$\text{For example } p = \begin{cases} +a_1 & \text{for most attractive} \\ +a_2 & \text{for attractive} \\ +a_3 & \text{for normal} \\ +a_4 & \text{for not so attractive} \\ +a_5 & \text{for least attractive} \end{cases}$$

where $\sum a_i = 1$ and a_i can be negative. (then positive number means favour and negative number means unfavour)

Thus in thus direction selection scheme, for example



The probability of direction selection of

1. the case with no pheromone : $\begin{cases} p_8 = a \\ p_{other} = \dfrac{1-a}{7} \end{cases}$

, where a is defined as above.

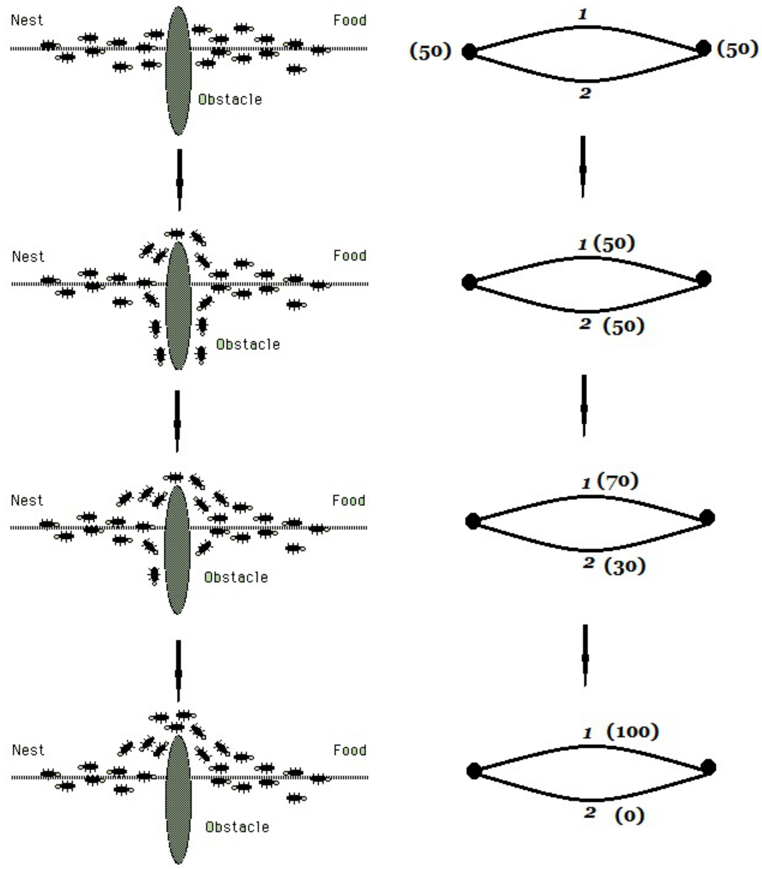
2. the upper case with pheromone

$$\begin{cases} p_1 = p_7 = \dfrac{b}{7} + a_4 \\ p_2 = p_6 = \dfrac{b}{7} + a_3 \\ p_3 = p_5 = \dfrac{b}{7} + a_2 \\ p_4 = \dfrac{b}{7} + a_1 \\ p_8 = a_5 - b \end{cases} \quad (\text{ notics that } \sum p_i = 1)$$

3. the lower case with pheromone

$$\begin{cases} p_3 = \dfrac{b}{7} + a_1 \\ p_2 = p_4 = \dfrac{b}{7} + a_2 \\ p_1 = p_5 = \dfrac{b}{7} + a_3 \\ p_6 = \dfrac{b}{7} + a_4 \\ p_7 = \dfrac{b}{7} + a_5 \\ p_8 = a_4 - b \end{cases}$$

Now consider a binary path selection problem to understanding this algorithm



Initially there are totally 100 ants, 50 ants in net and 50 ants in the food area.

There are 2 possible path, path 1 (shorter) , path 2 (longer).

Since it takes longer time to travel thru path2, thus pheromone in path 2 will keep dropping (by evapouration) and pheromone in path 1 will accumulate, and this process will favour more ants to choose path 1, thus the number of ants taking path 1 will increase, and drop for path 2.

Finally all ants will chose path 1 (the optimal path).

This example shows that the ACO method favours short path, thus this method can be used in solving the NP-hard *Travelling salesman problem*

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