Common glossaries in optimization

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What is this My personal collection of glossaries in optimization that I have seen 999 times, or my set of notations. This document will be kept updated.

1 Symbol

- k : iteration counter
- p^* : primal optimal value
- d^* : dual optimal value
- $\mathbf{x}^*$: an optimizer

2 Set

- $\mathbb{R}, \bar{\mathbb{R}}, \mathbb{R}_+, \mathbb{R}^n$: reals, extended reals (including ∞), nonnegative reals, n-dimensional reals
- $[n] = \{1, 2, \cdots, n\}$
- $\text{supp}(\mathbf{x}) = \{i \in [n] \mid x_i \neq 0\}$, for a given vector $\mathbf{x} \in \mathbb{R}^n$

3 Function

For a real-valued function $f : \mathbb{R}^n \to \mathbb{R}$,

- $f \in \mathcal{C}^0$: f is continuous
- $f \in \mathcal{C}^1$: f and gradient of f are continuous
- $f \in \mathcal{C}^2$: f, gradient of f and Hessian of f are continuous
- $f \in \mathcal{C}^1_L$: $f \in \mathcal{C}^1$ and gradient of f is L-Lipschitz, or f is L-smooth
- $f \in \mathcal{F}^k_L$: f is $\mathcal{C}^k_L$ and convex, or f is convex and L-smooth
- $f \in \mathcal{S}^k_{M,L}$: f is $\mathcal{F}^k_L$ and M-strongly convex
- f is L-Lipschitz if $|f(x) - f(y)| \leq L\|x - y\|$, $L \geq 0$
- f is nonexpansive if f is 1-Lipschitz
- f^* convex conjugate of f
- f is KL : f fulfill Kurdyka-Lojasiewicz inequality

4 Special functions

- $i_{\mathcal{C}}(\mathbf{x})$: indicator function of the set $\mathcal{C}$. $i_{\mathcal{C}}(\mathbf{x}) = 0$ if $\mathbf{x} \in \mathcal{C}$, $i_{\mathcal{C}}(\mathbf{x}) = \infty$ if $\mathbf{x} \notin \mathcal{C}$,
- $[\ \cdot\ ]_+ = \max\{\cdot, 0\}$.

5 Derivative of function

6 Duality

7 Shorthands

- nnz : number of non-zeros
- FOC : First order optimality condition / Fermat's rule
- NNLS : nonnegative least sqaures

8 Algorithm names

- GD : gradient descent
- PGD / ProjGD : projected gradient descent
- PGD / ProxGD : proximal gradient descent
- SG : Stochastic gradient
- HBM : Heavy ball method
- NAG : Nesterov's accelerated gradient
- FOM : First order methods
- ADMM : Alternating direction method of multipliers
- IPM : Interior point method
- AS : Active Set
- AA : Anderson's Acceleration

9 Algorithm properties

- Monotone : $f(\mathbf{x}_{k+1}) \leq f(\mathbf{x}_k)$