Servo Motor

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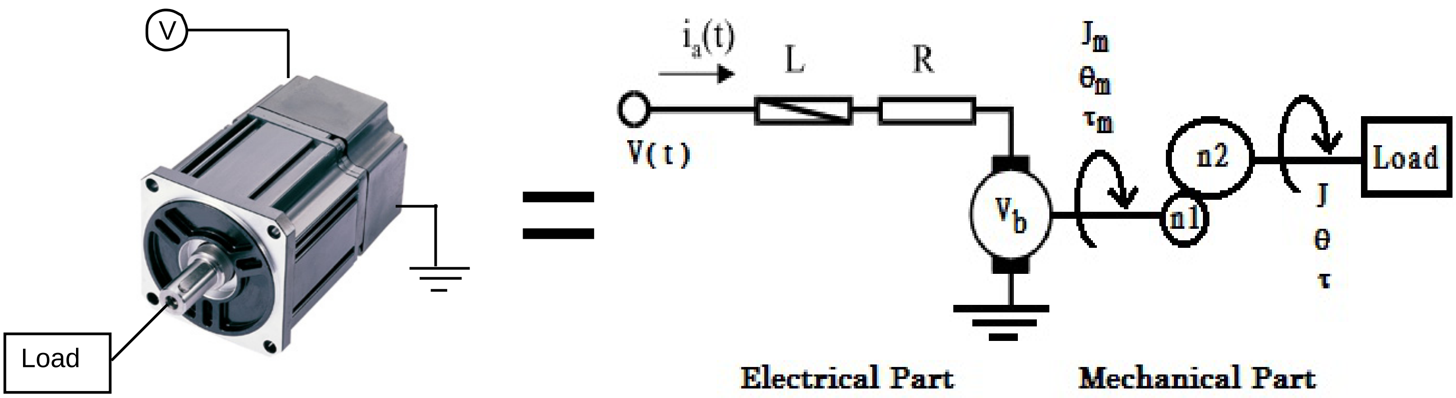

1 The Servo Motor System Function

First, an induction motor can be considered as LR Circuit

The Motor Torque τ_M is directly proportional to the motor ampeture current $i_A(t)$

And a back emf v_b produced in the motor is directly proportional to motor moving speed ω_m

Therefore $\qquad \tau_m(t) = K_m i_A(t) \qquad v_b(t) = K_b \omega_m(t) = K_b \dot{\theta}_m(t)$

(Note : More detail model is that $\tau = K_1 \phi i_A$ and $v_b = K_2 \phi \omega_m$, where ϕ is the magnetic flux inside the motor , which can be considered as a constant)

Therefore, apply the Kirchhoff's Voltage Law on the electrical part

$$v(t) = L\frac{di_A(t)}{dt} + Ri_A(t) + v_b(t)$$

Then consider the Mechanical part

The gear ratio $\eta = \dfrac{n_2}{n_1}$, and reduction gear ratio $\gamma = \dfrac{1}{\eta}$ is a magnification factor that reflect the load angle θ on the motor as

$$\theta = \eta \theta_m$$

And thus the load torque τ on the motor become

$$\tau \to \gamma \tau$$

Therefore, consider friction exists on the system, the rotational motion equation (Newton's 2nd Law) is thus

$$J_m \frac{d^2\theta_m(t)}{dt^2} + B\frac{d\theta_m(t)}{dt} = \tau_m - \gamma\tau$$

Therefore the 2 equation of the whole Servo Motor system are

$$\begin{cases} v(t) = L\dfrac{di_A(t)}{dt} + Ri_A(t) + v_b(t) \\ J_m \dfrac{d^2\theta_m(t)}{dt^2} + B\dfrac{d\theta_m(t)}{dt} = \tau_m - \gamma\tau \end{cases}$$

Apply Laplace Transform with zero initial condition

$$\begin{cases} v(s) = Lsi_A(s) + Ri_A(s) + v_b(s) \\ J_m s^2\theta_m(s) + Bs\theta_m(s) = \tau_m(s) - \gamma\tau(s) \end{cases}$$

$$\begin{cases} v(s) = (Ls + R)\, i_A(s) + v_b(s) \\ s\,(J_m s + B)\,\theta_m(s) = \tau_m(s) - \gamma\tau(s) \end{cases}$$

Apply the equation

$$\tau_m(t) = K_m i_A(t) \qquad v_b(t) = K_b\dot{\theta}_m(t)$$

$$\begin{cases} v(s) = (Ls + R)\, i_A(s) + K_b s\theta_m(s) \\ s\,(J_m s + B)\,\theta_m(s) = K_m i_A(s) - \gamma\tau(s) \end{cases}$$

Rearrange

$$\begin{cases} (Ls + R)\, i_A(s) = v(s) - K_b s\theta_m(s) \\ s\,(J_m s + B)\,\theta_m(s) = K_m i_A(s) - \gamma\tau(s) \end{cases}$$

These coupled equations can be expressed as one equation by eliminate the i_A term

$$s\,(J_m s + B)\,\theta_m(s) = K_m \frac{v(s) - K_b s\theta_m(s)}{Ls + R} - \gamma\tau(s)$$

$$s\,(Ls + R)\,(J_m s + B)\,\theta_m(s) = K_m v(s) - K_m K_b s\theta_m(s) - (Ls + R)\,\gamma\tau(s)$$

$$s\left[(Ls + R)\,(J_m s + B) + K_m K_b\right]\theta_m(s) = K_m v(s) - (Ls + R)\,\gamma\tau(s)$$

Since $\gamma = \dfrac{1}{\eta} = \dfrac{1}{\frac{n_2}{n_1}} = \dfrac{n_1}{n_2}$, by mechanical design, one can set η very large such that γ is very small, and thus $\gamma\tau(s) \approx 0$

Then

$$s\left[(Ls + R)\,(J_m s + B) + K_m K_b\right]\theta_m(s) = K_m v(s)$$

And the transfer function is thus

$$G_{\theta v}(s) = \frac{\theta_m(s)}{v(s)} = \frac{K_m}{s\left[(Ls + R)\,(J_m s + B) + K_m K_b\right]}$$

When $\gamma\tau \neq 0$, consider the transfer function between θ and τ (with $V = 0$)

$$s\left[(Ls + R)(J_m s + B) + K_m K_b\right]\theta_m(s) = -(Ls + R)\gamma\tau(s)$$

$$G_{\theta\tau}(s) = \frac{\theta_m(s)}{\tau(s)} = \frac{-(Ls + R)\gamma}{s\left[(Ls + R)(J_m s + B) + K_m K_b\right]}$$

2 Time Constant Simplification

Consider the following 2 time constant

$$\frac{L}{R} = t_E \qquad \text{(Electrical time constant)}$$

$$\frac{J_m}{B} = t_M \qquad \text{(Mechanical time constant)}$$

Usually $t_M \ggg t_E$, and therefore the following approximation can be used

$$\frac{L}{R} \approx 1$$

The equation

$$\begin{cases} G_{\theta v}(s) = \dfrac{\theta_m(s)}{v(s)} = \dfrac{K_m}{s\left[(Ls + R)(J_m s + B) + K_m K_b\right]} \\ G_{\theta\tau}(s) = \dfrac{\theta_m(s)}{\tau(s)} = \dfrac{-(Ls + R)\gamma}{s\left[(Ls + R)(J_m s + B) + K_m K_b\right]} \end{cases}$$

Can be simplified as

$$\begin{cases} G_{\theta v}(s) = \dfrac{\theta_m(s)}{v(s)} = \dfrac{K_m}{s\left[J_m s + B + \dfrac{K_m K_b}{R}\right]} \\ G_{\theta\tau}(s) = \dfrac{\theta_m(s)}{\tau(s)} = \dfrac{-\gamma}{s\left[J_m s + B + \dfrac{K_m K_b}{R}\right]} \end{cases}$$

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