Image Capture

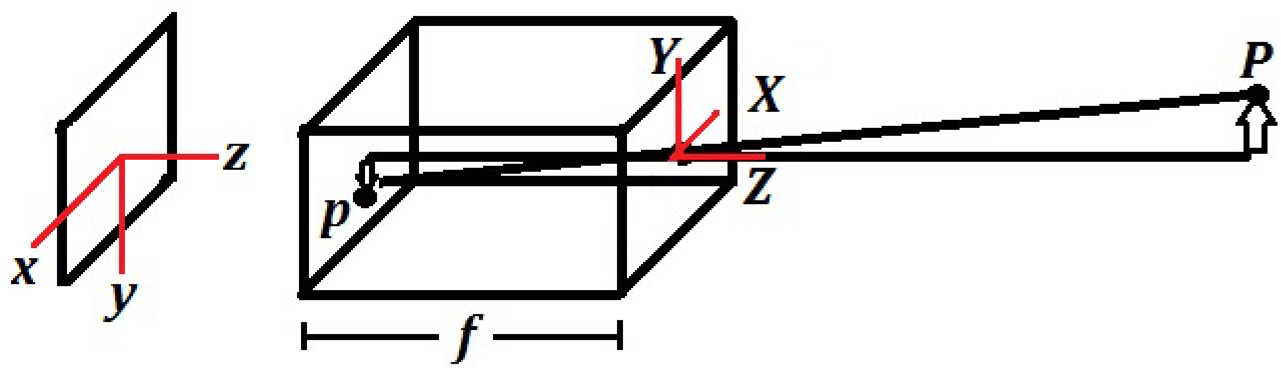
May 17, 2013

There are 4 coordinate

$P(X_0, Y_0, Z_0)$	3D object w.r.t world frame
$P(X_C, Y_C, Z_C)$	3D object w.r.t. camera frame
$p(x, y)$	2D image projection w.r.t focal plane
$p'(x', y')$	2D pixel projection w.r.t CCD

1 3D Camera Frame Object to 2D image projection

Consider the pinhole camera model



By simple geometry , the position of $P(X_C, Y_C, Z_C)$ and $p(x, y, f)$ is thus related by

$$\frac{X_C}{x} = \frac{Y_C}{y} = \frac{Z_C}{f}$$

Since the $z = f$ will not change, thus the object $P(X_C, Y_C, Z_C)$ in 3D camera frame is projected on focal plane $p(x, y)$ in 2D

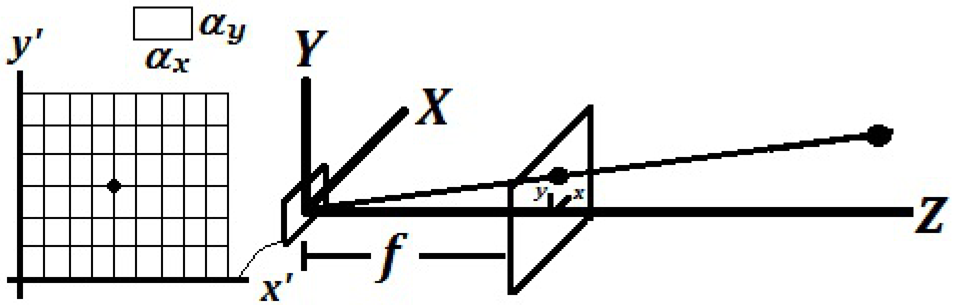
In matrix form

$$Z_C \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix}$$

Sometime it is convinent to put the focal plane infront of the camera

2 2D Image coordinate to 2D Pixel Frame

A Charge Coupled Device is to capture image in pixel frame.



α_x, α_y	scaling factor
c_x, c_y	Pixel center

i.e.

$$\text{if } \alpha_x = \alpha_y \text{ , the pixel is square pixel}$$

$$\begin{pmatrix} c_x \\ c_y \end{pmatrix} = \begin{pmatrix} \frac{x'_{max} + x'_{min}}{2} \\ \frac{y'_{max} + y'_{min}}{2} \end{pmatrix}$$

The transformation is

$$\begin{cases} x = \alpha_x \left(x' - c_x \right) \\ y = \alpha_y \left(y' - c_y \right) \end{cases}$$

In matrix form

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\alpha_x} & & c_x \\ & \frac{1}{\alpha_y} & c_y \\ & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

3 3D object in world frame to camera frame

If the object $P(X_0, Y_0, Z_0)$ is w.r.t world frame but not the camera frame (X_c, Y_c, Z_c)

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix} = H_0^C \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = \begin{pmatrix} & R_0^C & & d_0^C \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix} = \left(H_0^C \right)^{-1} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix} = \begin{pmatrix} & \left(R_0^C \right)^T & & \left(-R_0^C \right)^T d_0^C \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

4 Projection Matrix

$$P(X_0, Y_0, Z_0) \xrightarrow{H_0^C} P(X_C, Y_C, Z_C) \xrightarrow{f} p(x, y) \xrightarrow{c_{xy}, \alpha_{xy}} p'(x', y')$$

First consider the camera frame to focal plane transform

$$Z_C \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{pmatrix}$$

Then consider the world frame to camera frame transfrom

$$Z_0 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \left(R_0^C\right)^T & \left(-R_0^C\right)^T d_0^C \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

Next consider the focal plane to pixel plane transform

$$Z_0 \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\alpha_x} & & c_x \\ & \frac{1}{\alpha_y} & c_y \\ & & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \left(R_0^C\right)^T & \left(-R_0^C\right)^T d_0^C \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

Combine the first 2 matrix

$$Z_0 \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{f}{\alpha_x} & & c_x & 0 \\ & \frac{f}{\alpha_y} & c_y & 0 \\ & & 1 & 0 \end{pmatrix} \begin{pmatrix} \left(R_0^C\right)^T & \left(-R_0^C\right)^T d_0^C \\ 0 \quad 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

Since the last column of the first matrix are all zero

$$Z_0 \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{f}{\alpha_x} & & c_x \\ & \frac{f}{\alpha_y} & c_y \\ & & 1 \end{pmatrix}}_{Internal} \underbrace{\begin{pmatrix} \left(R_0^C\right)^T & \left(-R_0^C\right)^T d_0^C \end{pmatrix}}_{External} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{pmatrix}$$

And the equation above is the projection matrix

Parameters : f , c_x , c_y , α_x , α_y and external properties of H_0^C

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