First Order System

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The General First Order Transfer Function

$$G_I(s) = \frac{A_{DC}}{s\tau + 1}$$

A_{DC} : The DC gain $= G(0)$
τ : The Time constant

The Impulse Response

$$G_I(s) = \frac{A_{DC}}{s\tau + 1}$$

From the Laplace Analysis , the Impulse Response of First Order System is

$$g(t) = \mathcal{L}^{-1}\left\{G_I(s)\right\} = \mathcal{L}^{-1}\left\{\frac{A_{DC}}{s\tau + 1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{A_{DC}}{s\tau + 1}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{A_{DC}}{\tau}}{s + \frac{1}{\tau}}\right\} = \frac{A_{DC}}{\tau} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s + \frac{1}{\tau}}\right\} = \frac{A_{DC}}{\tau} e^{-\frac{1}{\tau}t} = \frac{A_{DC}}{\tau} e^{-\frac{t}{\tau}}$$

The Step Response

From the Laplace Analysis , the Step Response of First Order System is

$$g(t) = \mathcal{L}^{-1}\left\{\frac{G_I(s)}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{A_{DC}}{s\tau + 1}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{A_{DC}}{\tau}}{s\left(s + \frac{1}{\tau}\right)}\right\}$$

The Partial Fraction Decomposition via Heaviside Cover Up Method

$$\frac{\frac{A_{DC}}{\tau}}{s\left(s + \frac{1}{\tau}\right)} = \frac{A_{DC}}{\tau} \frac{1}{s\left(s + \frac{1}{\tau}\right)} = \frac{A_{DC}}{\tau}\left(\frac{\left[\frac{1}{s + 1/\tau}\right]_{s=0}}{s} + \frac{\left[\frac{1}{s}\right]_{s=-\frac{1}{\tau}}}{s + 1/\tau}\right)$$

$$= \frac{A_{DC}}{\tau}\left(\frac{\tau}{s} + \frac{-\tau}{s + 1/\tau}\right) = A_{DC}\left(\frac{1}{s} - \frac{1}{s + 1/\tau}\right)$$

Thus the Step Response is

$$g(t) = \mathcal{L}^{-1}\left\{A_{DC}\left(\frac{1}{s} - \frac{1}{s + 1/\tau}\right)\right\} = A_{DC}\left(1 - e^{-\frac{t}{\tau}}\right)$$

The Ramp Response

From the Laplace Analysis , the Ramp Response of First Order System is

$$g(t) = \mathcal{L}^{-1}\left\{\frac{G_I(s)}{s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\cdot\frac{A_{DC}}{s\tau+1}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{A_{DC}}{\tau}}{s^2\left(s+\frac{1}{\tau}\right)}\right\}$$

Partial Fraction Decomposition

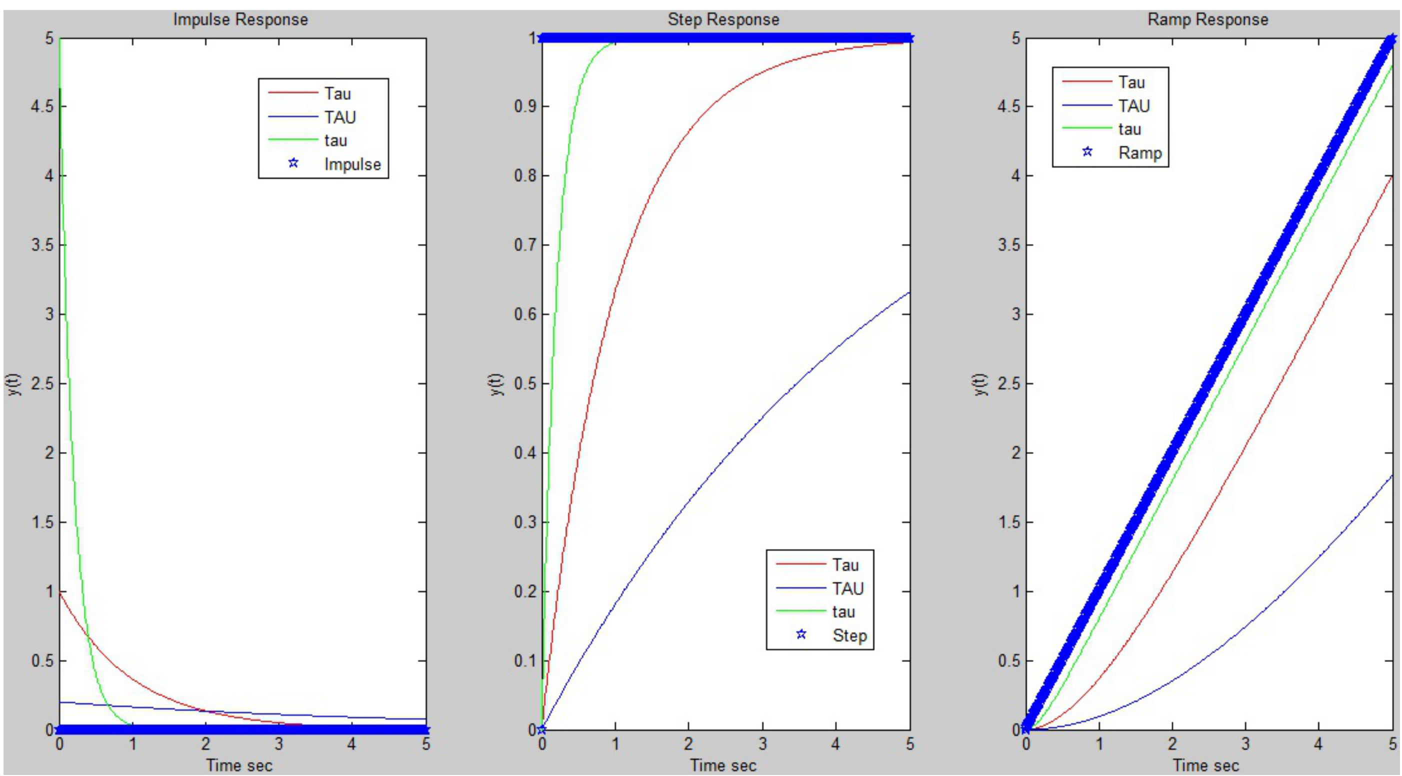
$$\frac{\frac{A_{DC}}{\tau}}{s^2\left(s+\frac{1}{\tau}\right)} = \frac{A_{DC}}{\tau}\left[\frac{1}{s^2\left(s+{}^1/_\tau\right)}\right] = \frac{A_{DC}}{\tau}\left[\frac{\left(\frac{d}{ds}\frac{1}{s+{}^1/_\tau}\right)_{s=0}}{s} + \frac{\left(\frac{1}{s+{}^1/_\tau}\right)_{s=0}}{s^2} + \frac{\left(\frac{1}{s^2}\right)_{s=-\frac{1}{\tau}}}{s+{}^1/_\tau}\right]$$

$$= \frac{A_{DC}}{\tau}\left[\frac{-\tau^2}{s} + \frac{\tau}{s^2} + \frac{\tau^2}{s+{}^1/_\tau}\right] = A_{DC}\left[\frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+{}^1/_\tau}\right] = A_{DC}\left[\frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+{}^1/_\tau}\right]$$

Thus the Ramp Response is

$$g(t) = \mathcal{L}^{-1}\left\{A_{DC}\left[\frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+{}^1/_\tau}\right]\right\} = A_{DC}\left[-\tau + t + \tau e^{-\frac{t}{\tau}}\right] = A_{DC}\left[t - \tau + \tau e^{-\frac{t}{\tau}}\right]$$

The Plots



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