Block Algebra & Steady-State Value

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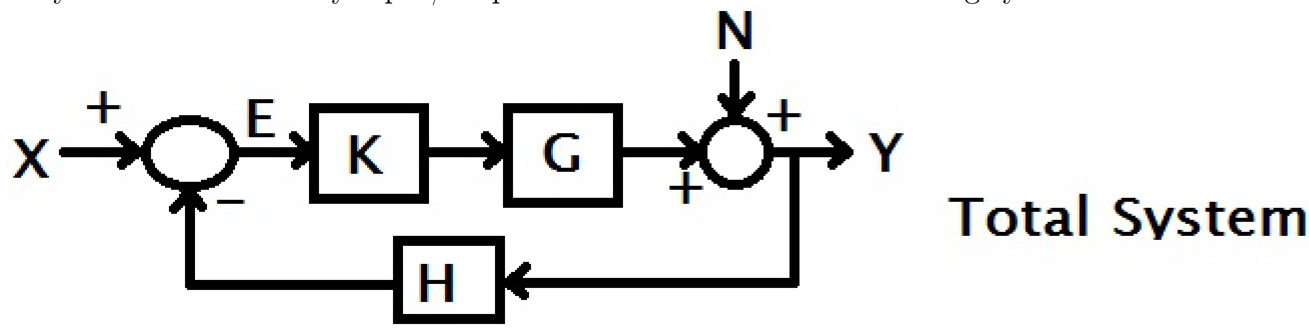
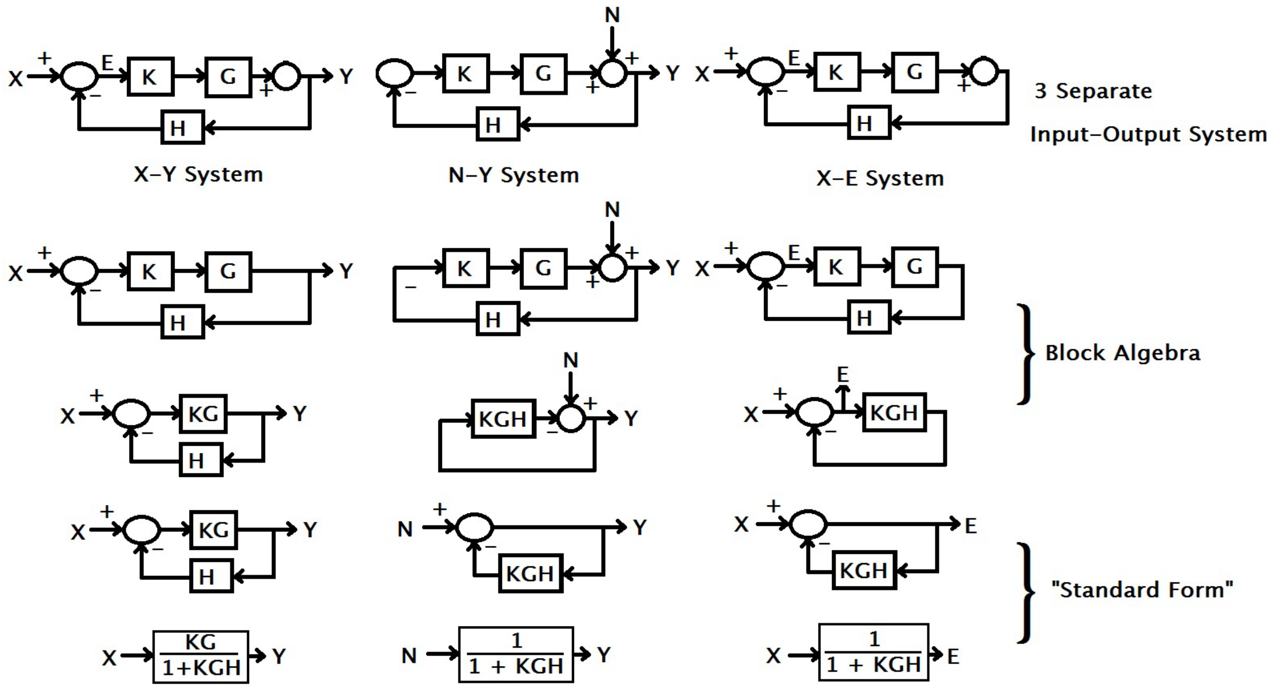
A system can have many input/output relation. Consider the following system that



- Standard input X
- Standar output Y
- Controller K
- Plant G
- Noise N
- Feedback component H
- Error term E

The system can be treated as 3 system : $X - Y$, $X - E$ and $N - Y$
For sure it can be consider as $N - E$, $E - Y$, etc.



Then

For the $X - Y$ block

$$\frac{Y(s)}{X(s)} = G_{XY}(s) = \frac{K(s)G(s)}{1 + K(s)G(s)H(s)}$$

Thus, the steady-state output can be found by using Final Value Theorem of Laplace Transfrom

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\mathcal{L}\left\{X(s)\right\}$$

Thus

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} sG(s)X(s)$$

For step input :$X(s) = \dfrac{1}{s}$, the steady-state output is thus

$$y(\infty) = G_{XY}(0)$$

Using similar logic, the steady state error can be found by

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_{XE}(s)X(s)$$

For step input, the steady state error is thus

$$e(\infty) = G_{XE}(0) = \frac{1}{1 + K(0)G(0)H(0)}$$

$-END-$