PID Control

Andersen Ang

First created: 2013. Last update: 2017-Feb-1

PID Control stand for Proportional, Integral , Differential Control

$$K_P + K_I \frac{1}{s} + K_D s$$

K_P , K_I , K_D are the gain parameter of the controller.

1 P

Consider the original system with OLTF (Open-Loop Transfer Function) as

$$G_{OL}(s) = \frac{1}{s+p}$$

Thus the CLTF (Close-Loop Transfer Function) and is thus

$$G_{CL}(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{1}{s+p}}{1+\frac{1}{s+p}} = \frac{1}{s+(p+1)}$$

Corresponding Impulse Response is thus

$$g(t) = \mathcal{L}^{-1}\left\{G_{CL}(s)\right\} = e^{-(p+1)t}$$

Now consider the OLTF with P-Controller

$$G'(s) = \frac{K_P}{s+p}$$

Thus the corresponding CLTF and impulse response is

$$G'_{CL}(s) = \frac{K_P}{s+(p+1+K_P)} \qquad g_{CL}(t) = K_p e^{-(p+1+K_p)t}$$

2 D

Consider the original system with OLTF (Open-Loop Transfer Function) as

$$G_{OL}(s) = \frac{1}{s+p}$$

Now consider the OLTF with D-Controller

$$G'(s) = \frac{K_D s}{s+p}$$

Thus the corresponding CLTF is thus

$$G_{CL}(s) = \frac{K_D s}{(K_D+1)s+p}$$

The impulse respond

$$g_{CL}(t) = \frac{K_D}{K_D+1}\delta(t) - \frac{p}{(K_D+1)^2}\exp\left(-\frac{pt}{K_D+1}\right)$$

3 I

Consider the original system with OLTF (Open-Loop Transfer Function) as

$$G_{OL}(s) = \frac{1}{s+p}$$

Now consider the OLTF with I-Controller

$$G'(s) = \frac{K_I}{s(s+p)}$$

Perform the Partial Fraction by Heaviside Cover up method

$$G'(s) = \frac{\frac{K_I}{p}}{s+p} - \frac{\frac{K_I}{p}}{s}$$

Thus the corresponding CLTF is thus

$$G_{CL}(s) = \frac{\frac{K_I}{p}}{s^2+ps+\frac{K_I}{p}}$$

The impulse respond can be solved by referring to 2nd order system.

4 PID

Consider the original system with OLTF (Open-Loop Transfer Function) as

$$G_{OL}(s) = \frac{1}{s+p}$$

Now consider the OLTF with PID-Controller

$$G'(s) = \frac{1}{s+p}\left(K_P + K_I\frac{1}{s} + K_D s\right)$$

The effect will be the sum of all three controller added together.

$-END-$