Directivity

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1 Direction dependence of radiation fields

Recall that the total fields of Hertzian Dipole are

$$\widetilde{\mathbf{H}} = \frac{I_0 dl}{4\pi} \sin\theta e^{-j\beta r} \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) \hat{\phi}$$

$$\widetilde{\mathbf{E}} = \eta \frac{I_0 dl}{2\pi} \cos\theta e^{-j\beta r} \left(\frac{1}{r^2} - \frac{j}{\beta r^3} \right) \hat{r} + \eta \frac{I_0 dl}{4\pi} \sin\theta e^{-j\beta r} \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right] \hat{\theta}$$

Consider

$$\begin{cases} \text{Near Field} \begin{cases} \widetilde{\mathbf{H}}(r,\theta) = \dfrac{I_0 dl}{4\pi r^2} \sin\theta e^{-j\beta r} \hat{\phi} \\ \\ \widetilde{\mathbf{E}}(r,\theta) = j\dfrac{\eta}{\beta} \dfrac{I_0 dl}{2\pi r^3} \cos\theta e^{-j\beta r} \hat{r} - j\dfrac{\eta}{\beta} \dfrac{I_0 dl}{4\pi r^3} \sin\theta e^{-j\beta r} \hat{\theta} \end{cases} \\ \\ \text{Far Field} \begin{cases} \widetilde{\mathbf{H}}(r,\theta) = j\dfrac{\beta I_0 dl}{4\pi r} \sin\theta e^{-j\beta r} \hat{\phi} \\ \widetilde{\mathbf{E}}(r,\theta) = j\dfrac{\eta\beta I_0 dl}{4\pi r} \sin\theta e^{-j\beta r} \hat{\theta} \end{cases} \end{cases}$$

In time domain

$$\begin{cases} \text{Near Field} \begin{cases} \mathbf{H}(r,\theta,t) = \dfrac{I_0 dl}{4\pi r^2} \cos(\omega t - \beta r) \sin\theta \hat{\phi} \\ \\ \mathbf{E}(r,\theta,t) = -\dfrac{\eta}{\beta} \dfrac{I_0 dl}{2\pi r^3} \sin(\omega t - \beta r) \cos\theta \hat{r} + \dfrac{\eta}{\beta} \dfrac{I_0 dl}{4\pi r^3} \sin(\omega t - \beta r) \sin\theta \hat{\theta} \end{cases} \\ \\ \text{Far Field} \begin{cases} \mathbf{H}(r,\theta,t) = -\dfrac{\beta I_0 dl}{4\pi r} \sin(\omega t - \beta r) \sin\theta \hat{\phi} \\ \mathbf{E}(r,\theta,t) = -\dfrac{\eta\beta I_0 dl}{4\pi r} \sin(\omega t - \beta r) \sin\theta \hat{\theta} \end{cases} \end{cases}$$

Now recall that the time domain plane wave

$$\begin{cases} \mathbf{H}(r,t) = \dfrac{E_0}{\eta}\cos(\omega t - \beta z)\,\hat{x} \\ \mathbf{E}(r,t) = E_0 \cos(\omega t - \beta z)\,\hat{y} \end{cases}$$

- Notice that compare to plane wave that generally expressed as $f(r,t)$, the radiation fields from also depends on θ
- The fields will have different magnitude at different θ(angle of elevation) , and this property is called directivity of antenna.
- For the Hertzian dipole, the directivity factor is $\sin\theta$
- More general, the field directivity is depended on both elevation and azimuthal angle , thus general directivity factor is a function of θ and ϕ
- Therefore, for antenna, the fields can be generally expressed as

$$E(r,t,\theta,\phi) = E(r,t)f(\theta,\phi)$$

- where $f(\theta,\phi)$ is the directivity factor, or called unit pattern
- That means, in the case of Hertzian dipole, the unit pattern $f(\theta,\phi) = \sin\theta$, it is independent of azimuthal angleϕ
- Unlike the fields of plane wave that the field intensity is independent of direction, the directivity of antenna fields simply means that

orientation of antenna need to be adjusted to achieve max transmitting property

2 Field Pattern

Consider the following normalization

$$F(\theta,\phi) = \frac{f(\theta,\phi)}{\max f(\theta,\phi)}$$

Then

$$\max F(\theta,\phi) = 1$$

And therefore

$$|E| = |E(r,t)|F(\theta,\phi)$$

- For Hertzian dipole, $F = f$ since $\max f = \max \sin\theta = 1$
- Direction with max radiation : major direction
- Direction without radiation : null direction
- Radiation lobe of major direction : Main lobe

3 Antenna Gain and Directivity

If the antenna is lossy (not perfect), the efficiency η (not to confuse with intrinsic impedance)

$$\eta = \frac{P_{Rad}}{P_{In}} \in [0, 1]$$

If antenna is perfect , $\eta = 1$

The antenna gain is the ratio of radiation power to average radiation power

$$G = \frac{\left\langle \bar{S}(r) \right\rangle}{P^{Avg}} = \frac{\left\langle \bar{S}(r) \right\rangle}{P_{Rad} / 4\pi r^2}$$

The directivity D is defined as the max gain

$$D = \max G$$

That is

$$D = \max G = 4\pi r^2 \frac{\left\langle \bar{E}(r) \times \bar{H}(r) \right\rangle}{P_{Rad}}$$

Consider the E-field and H-field expression as

$$\widetilde{\mathbf{H}} = \frac{I_0 dl}{4\pi} \sin\theta e^{-j\beta r} \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) \hat{\phi}$$

$$\widetilde{\mathbf{E}} = \eta \frac{I_0 dl}{2\pi} \cos\theta e^{-j\beta r} \left(\frac{1}{r^2} - \frac{j}{\beta r^3} \right) \hat{r} + \eta \frac{I_0 dl}{4\pi} \sin\theta e^{-j\beta r} \left[\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right] \hat{\theta}$$

The directivity of Hertzian dipole antenna is thus (after some simplification):

$$D = \max \frac{e^{-j\beta r} \eta I_0 r dl}{P_{Rad}} \left\langle \left(2\cos\theta \left(\frac{1}{r} - \frac{j}{\beta r^2} \right) \hat{r} + \sin\theta \left(j\beta + \frac{1}{r} - \frac{j}{\beta r^2} \right) \hat{\theta} \right) \times \left(\sin\theta \left(j\beta + \frac{1}{r} \right) \hat{\phi} \right) \right\rangle$$

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