Some points on electromagnetic and electronics

Andersen Ang

First created: before 2013. Last updated: 2017-Feb-1

1 Vector Calculus

1. Differential Length $d\bar{l} = dx\hat{x} + dy\hat{y} + dz\hat{z} = dr\hat{r} + rd\phi\hat{\phi} + dz\hat{z} = dR\hat{R} + Rd\theta\hat{\theta} + R\sin\theta d\phi\hat{\phi}$
2. Differential normal surface $d\mathbf{S} = \begin{cases} dydz\hat{x} \\ dzdx\hat{y} \\ dxdy\hat{z} \end{cases} = \begin{cases} rd\phi dz\hat{r} \\ drdz\hat{\phi} \\ rdrd\phi\hat{z} \end{cases} = \begin{cases} R^2\sin\theta d\theta d\phi\hat{r} \\ R\sin\theta drd\phi\hat{\theta} \\ RdRd\theta\hat{\phi} \end{cases}$
3. Differential volume $dV = dxdydz = rdrd\phi dz = r^2\sin\theta drd\theta d\phi$
4. Line integral work done$\int_C \bar{V}\cdot d\bar{l} = \int_a^b |\bar{V}|\cos\theta dl$
5. Flux $\Psi = \iint_S \bar{V}\cdot d\bar{S} = \iint_S |\bar{V}|\cos\theta dS$
6. Volume $V = \iiint_V \rho_v dV$
7. Directional derivative $\left.\dfrac{\partial\Phi}{\partial l}\right|_p = \lim\limits_{\Delta l\to 0}\dfrac{\Phi(p')-\Phi(p)}{\Delta l}$
8. Definition of gradient $grad\,\Phi = \dfrac{\partial\Phi}{\partial x}\hat{x} + \dfrac{\partial\Phi}{\partial y}\hat{y} + \dfrac{\partial\Phi}{\partial z}\hat{z} = \nabla\Phi$.
   Meaning : direction with maximum rate of change per unit distance
9. Directional derivative $\nabla\Phi\cdot\hat{a}$ = rate of change of Φ along direction of $\hat{a}$
10. Definition of Divergence $div\bar{V} = \lim\limits_{\Delta V\to 0}\left(\dfrac{1}{\Delta V}\iint_S \bar{V}\cdot d\bar{S}\right) = \nabla\cdot\bar{V}$.
    Meaning : outward flux per unit volume
11. Definition of Curl $curl\bar{V} = \lim\limits_{\Delta S\to 0}\left(\dfrac{1}{\Delta S}\left|\oint_C \bar{V}\cdot d\bar{l}\right|_{max}\right)\hat{n} = \nabla\times\bar{V}$.
    Meaning : circulation per unit area
12. Del / Nabla Operator :

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z} = \frac{\partial}{\partial R}\hat{R} + \frac{1}{R}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}$$

13. Gauss's Divergence Theorem : $\iiint_V \nabla \cdot \bar{V} dxdydz = \iint_{\partial V} \bar{V} \cdot d\bar{S}$

14. Classical Stoke's Theorem : $\iint_S \nabla \times \bar{V} \cdot d\bar{S} = \oint_{\partial S} \bar{V} \cdot d\bar{l}$

15. Definition of Divergence + Gauss Divergence Theorem

$$div\bar{V} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \iint \iint_S \bar{V} \cdot d\bar{S} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \iiint_V \nabla \cdot \bar{V} dxdydz = \nabla \cdot \bar{V}$$

16. Definition of Curl + Stoke's Theorem

$$curl\bar{V} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \left| \oint_C \bar{V} \cdot d\bar{l} \right|_{max} \hat{n} = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \iint_S \nabla \times \bar{V} \cdot d\bar{S} = \nabla \times \bar{V}$$

17. Laplacian of Scalar Field

$$\nabla \cdot \nabla \Psi = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \cdot \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \Psi$$
$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

18. Laplacian of Vector Field using (1) grad of div - curl of cur, (2) Scalar Laplacian

$$\nabla^2 \bar{V} = \nabla(\nabla \cdot \bar{V}) - \nabla \times \nabla \times \bar{V}$$
$$\nabla^2 \bar{V} = \nabla^2 V_x \hat{x} + \nabla^2 V_y \hat{y} + \nabla^2 V_z \hat{z}$$

19. Solenoidal Field : Div of curl = 0
$\iff$ rotational field is divergenceless $\iff$ No source / sink of flux
$\iff \nabla \cdot \underbrace{\nabla \times \bar{V}}_{\mathbf{F}} = 0$
$\iff \nabla \cdot \mathbf{F} = 0 \iff \iint_S \mathbf{F} \cdot d\mathbf{S} = \mathbf{0}$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_X & V_Y & V_Z \end{vmatrix} = \frac{\partial}{\partial x} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_Y & V_z \end{vmatrix} - \frac{\partial}{\partial y} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ V_X & V_Z \end{vmatrix} + \frac{\partial}{\partial z} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ V_X & V_Y \end{vmatrix} = 0$$

20. Irrotational field : Curl of grad = 0
$\iff$ gradient field is irrotational $\iff$ curl-free
$\iff \nabla \times \underbrace{\nabla \Psi}_{-\mathbf{F}} = 0 \iff \nabla \times \mathbf{F} = 0 \iff \oint_C \mathbf{F} \cdot \mathbf{dl} = 0$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \left\langle \frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\partial \Psi}{\partial z} \right\rangle = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \Psi}{\partial x} & \frac{\partial \Psi}{\partial y} & \frac{\partial \Psi}{\partial z} \end{vmatrix} = 0$$

21. Helmholtz Decomposition / Fundamental Theorem of Vector Calculus
$\bar{f} = -\nabla\phi + \nabla \times \bar{A}$, all decaying vector field f can be decomposed into an irrotational (curl-free) vector field and a solenoidal (divergence-free) vector field

–END–