Transisent Bouncing

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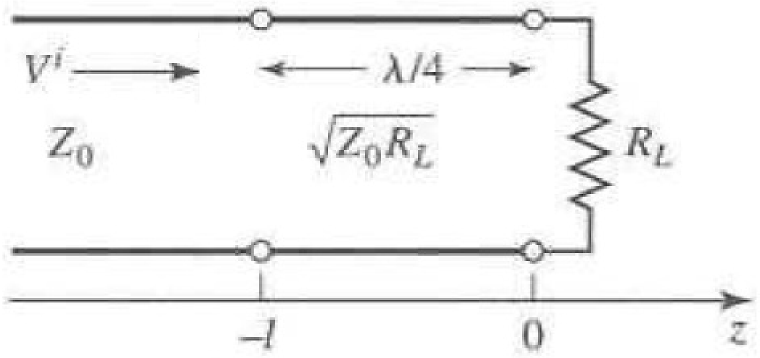
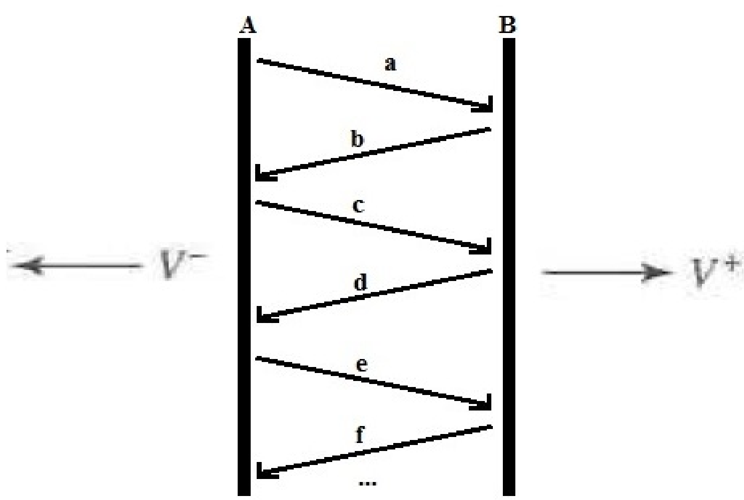
Before the wave settle down on the boundary, there is a transistent bouncing phenomenon.

Consider the diagram





The input wave V_{In} will be splitted into 2 wave on the boundary, $V^-$and V^+

Consider $a = V_{In}$ then

$$b = V_{In}\Gamma_B$$
$$c = V_{In}\Gamma_B\Gamma_A$$
$$d = V_{In}\Gamma_B^2\Gamma_A$$
$$e = V_{In}\Gamma_B^2\Gamma_A^2$$
$$f = V_{In}\Gamma_B^3\Gamma_A^2$$

$$ForwardWave = a + c + e + ... = V_{In}\left(1 + \Gamma_A\Gamma_B + (\Gamma_A\Gamma_B)^2 + ...\right) = \frac{V_{In}}{1 - \Gamma_A\Gamma_B}$$

$$BackwardWave = b + d + f + ... = -V_{In}\Gamma_B\left(1 + \Gamma_A\Gamma_B + (\Gamma_A\Gamma_B)^2 + ...\right) = \frac{-V_{In}\Gamma_B}{1 - \Gamma_A\Gamma_B}$$

Thus

$$V^{+} = \frac{V_{In}}{1-\Gamma_A\Gamma_B} \qquad V^{-} = \frac{-V_{In}\Gamma_B}{1-\Gamma_A\Gamma_B}$$

Where

$$\Gamma_A = \frac{\sqrt{Z_0R_L}-Z_0}{\sqrt{Z_0R_L}+Z_0} \qquad \Gamma_B = \frac{R_L-\sqrt{Z_0R_L}}{R_L+\sqrt{Z_0R_L}}$$

Thus

$$(1-\Gamma_A\Gamma_B)^{-1} = \left(1-\frac{\sqrt{Z_0R_L}-Z_0}{\sqrt{Z_0R_L}+Z_0}\frac{R_L-\sqrt{Z_0R_L}}{R_L+\sqrt{Z_0R_L}}\right)^{-1}$$

$$= \left[\frac{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)-\left(\sqrt{Z_0R_L}-Z_0\right)\left(R_L-\sqrt{Z_0R_L}\right)}{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)}\right]^{-1}$$

$$= \frac{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)}{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)-\left(\sqrt{Z_0R_L}-Z_0\right)\left(R_L-\sqrt{Z_0R_L}\right)}$$

$$= \frac{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)}{4Z_0R_L}$$

And

$$V_{In} = V^IT_A = V^I\left(1-\Gamma_A\right) = \frac{2Z_0V^I}{\sqrt{Z_0R_L}+Z_0}$$

Thus

$$V^{+} = \frac{V_{In}}{1-\Gamma_A\Gamma_B} = \underbrace{\frac{2Z_0V^I}{\sqrt{Z_0R_L}+Z_0}}_{V_{In}} \cdot \underbrace{\frac{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)}{4Z_0R_L}}_{(1-\Gamma\Gamma)^{-1}} = V^I\frac{R_L+\sqrt{Z_0R_L}}{2R_L}$$

$$V^{-} = \frac{-V_{In}\Gamma_B}{1-\Gamma_A\Gamma_B} = \underbrace{\frac{-2Z_0V^I}{\sqrt{Z_0R_L}+Z_0}}_{V_{In}} \cdot \underbrace{\frac{R_L-\sqrt{Z_0R_L}}{R_L+\sqrt{Z_0R_L}}}_{\Gamma B} \cdot \underbrace{\frac{\left(\sqrt{Z_0R_L}+Z_0\right)\left(R_L+\sqrt{Z_0R_L}\right)}{4Z_0R_L}}_{(1-\Gamma\Gamma)^{-1}} = V^I\frac{-R_L+\sqrt{Z_0R_L}}{2R_L}$$

$-END-$