Circles on Smith Chart

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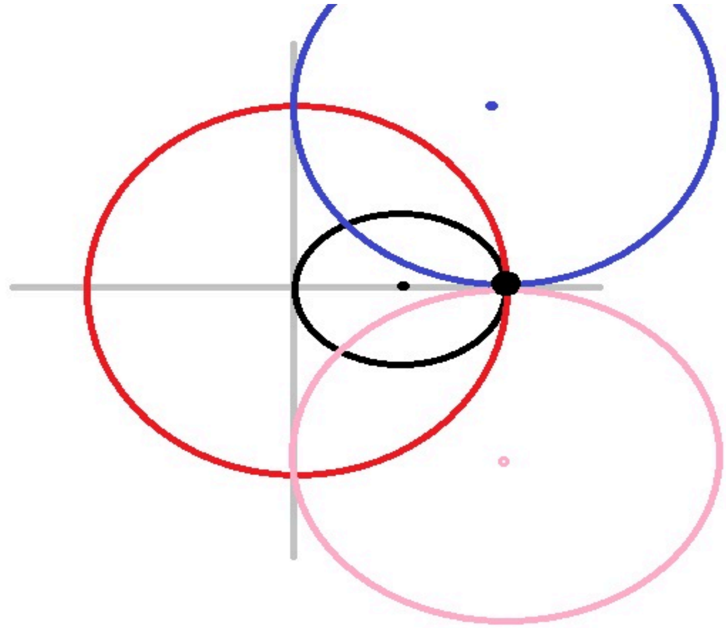

$$\left(\Gamma_R - \frac{z_R}{1+z_R}\right)^2 + \Gamma_I^2 = \left(\frac{1}{1+z_R}\right)^2$$

$$(\Gamma_I - 1)^2 + \left(\Gamma_I - \frac{1}{z_I}\right)^2 = \left(\frac{1}{z_I}\right)^2$$

Consider the first circle equation

When $z_R = 1$

$$\left(\Gamma_R - \frac{1}{2}\right)^2 + \Gamma_I^2 = \left(\frac{1}{2}\right)^2 \qquad \text{(The black circle)}$$

When $z_R = 0$

$$\Gamma_R^2 + \Gamma_I^2 = 1 \qquad \text{(The red circle)}$$

When $z_R = \infty$

$$(\Gamma_R - 1)^2 + \Gamma_I^2 = 0 \qquad \text{(The black dot)}$$

When $z_R = -1$

$$(\Gamma_R - \infty)^2 + \Gamma_I^2 = \infty \qquad \text{(Circle at infinity)}$$

Consider the second circle equation

When $z_I = 1$

$$(\Gamma_I - 1)^2 + (\Gamma_I - 1)^2 = 1 \qquad \text{(The blue circle)}$$

When $z_I = -1$

$$(\Gamma_I - 1)^2 + (\Gamma_I + 1)^2 = 1 \qquad \text{(The pink cicle)}$$

When $z_I = \infty$

$$(\Gamma_I - 1)^2 + \Gamma_I^2 = 0 \qquad \text{(The black dot)}$$

When $z_I = 0$

$$(\Gamma_I - 1)^2 + (\Gamma_I - \infty)^2 = \infty \qquad \text{(Circle at infinity)}$$

Different points on smith chart

Since this is the polar plot , thus :

$$\begin{cases} \text{points above x-axis : +ve reactance, inductor} \\ \text{points under x-axis : -vereactance, capacitor} \end{cases}$$

And the unit circle is $\Gamma = 1$

$z_L = 0$, Γ =-1 $z_L = 1$, Gamma=0 , $z_L = \infty$, Gamma = +1

Travel along the circle contour

$\Gamma(z) = \Gamma_0 e^{j2\beta(z-l)} = |\Gamma_0| e^{j\theta} e^{j2\beta(z-l)}$, θ is the phase angle of Γ_0 and $2\beta(z-l)$ is the angle along the constant circle

When $2\beta(z-l) = -2\pi$ (move anti-clockwise)

$2\beta(z-l) = -2\pi \iff 2\frac{2\pi}{\lambda}(z-l) = 2\pi \iff z - l = -\frac{\lambda}{2} \iff l - z = \frac{\lambda}{2}$

i.e. Anti-clockwise rotation around the smith chart is equal to half wavelength

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