Quarter Wavelength Transformer

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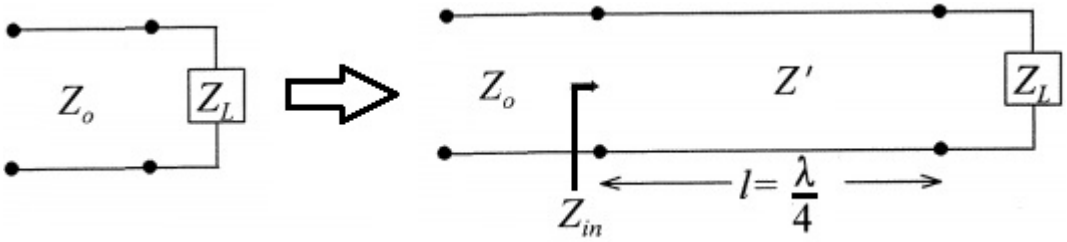
$\frac{\lambda}{4}$ transformer is a matching technique to eliminate reflection in transmission line

Recall the reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Match : $\Gamma = 0 \iff Z_L = Z_0$
Mismatch : $|\Gamma| > 0 \iff Z_L \neq Z_0$



When inserting a $\frac{\lambda}{4}$ length line with characteristic impedance Z'

The input impedance looking into the quarter wavelength transformer (assume lossless) is thus

$$Z_{In} = Z' \frac{Z_L + jZ' \tan \beta l}{Z' + jZ_L \tan \beta l}$$

When $l = \frac{\pi}{4}$

$$\beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \qquad \to \tan \beta l \to \infty$$

$$Z_{In} = Z' \frac{Z'}{Z_L} = \frac{Z'^2}{Z_L}$$

For matching, $Z_{In} = Z_0$

$$Z_{In} = \frac{Z'^2}{Z_L} = Z_0$$

i.e.

$$Z' = \sqrt{Z_0 Z_L}$$

Thus we can insert a transmission line with selected Z' to match the circuit to achieve no reflection

$$-END-$$