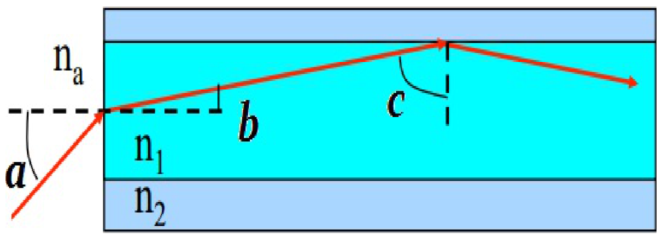
Refractive Index Requirement in Fibre Optics

Snell's Law Review

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \frac{c_0\sqrt{\mu_2\varepsilon_2}}{c_0\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \quad (\text{usually } \mu_2 = \mu_1 \approx \mu_0)$$

Consider the fibre



n_a : refractive index of air
n_1 : refractive index of material 1
n_2 : refractive index of material 2
Notice that $b + c = 90^o$

First $\frac{\sin a}{\sin b} = \frac{n_1}{n_a}$, then

$$\sin b = \frac{n_a}{n_1}\sin a \approx \frac{1}{n_1}\sin a \quad (n_a \approx 1)$$

Then for the fibre to work, $c > \theta_c$ (critical angle)

$$\sin c > \sin\theta_c$$

Since $b + c = 90^o$

$$\sin c = \sin(90 - b) = \cos b$$

i.e.

$$\sin c = \cos b > \sin\theta_c$$

$\cos b$ can be expressed as $\sin b$

$$\cos b = \sqrt{1 - \sin^2 b}$$

$$\sin c = \sqrt{1 - \sin^2 b} > \sin\theta_c$$

Applying Snell's Law on $\sin b$

$$\sin c = \sqrt{1 - \left(\frac{1}{n_1}\sin a\right)^2} > \sin\theta_c$$

Refraction between material 1 and 2

$$\sin\theta_c = \frac{n_2}{n_1}$$

Thus

$$\sqrt{1 - \left(\frac{1}{n_1}\sin a\right)^2} > \frac{n_2}{n_1}$$

$$1 - \frac{1}{n_1^2}\sin^2 a > \frac{n_2^2}{n_1^2}$$

Assume $n_2 = 1$

$$n_1^2 - \sin^2 a > 1$$

Therefore

$$n_1 > \sqrt{1 + \sin^2 a}$$

$\max\sin^2 a = 1$, so

$$n_1 > \sqrt{2} \approx 1.41$$

When $n_2 \neq 1$

$$1 - \frac{1}{n_1^2}\sin^2 a > \frac{n_2^2}{n_1^2}$$

Consider $\max\sin^2 a = 1$

$$1 - \frac{1}{n_1^2} > \frac{n_2^2}{n_1^2}$$

$$n_1^2 > n_2^2 + 1$$

$$n_1 > \sqrt{1 + n_2^2}$$

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