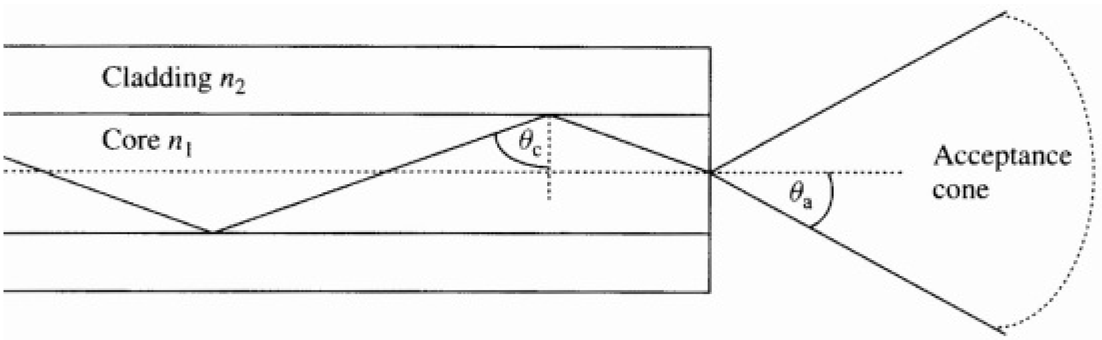
Fundamentals of Fibers

September 18, 2013

Numerical Aperture



Consider the diagram above, the acceptance angle $\theta_a = 90 - \theta_c$, thus

$$\sin\theta_a = \sin(90 - \theta_c) = \cos\theta_c = \sqrt{1 - \sin^2\theta_c}$$

Since

$$\sin\theta_c = \frac{n_2}{n_1}$$

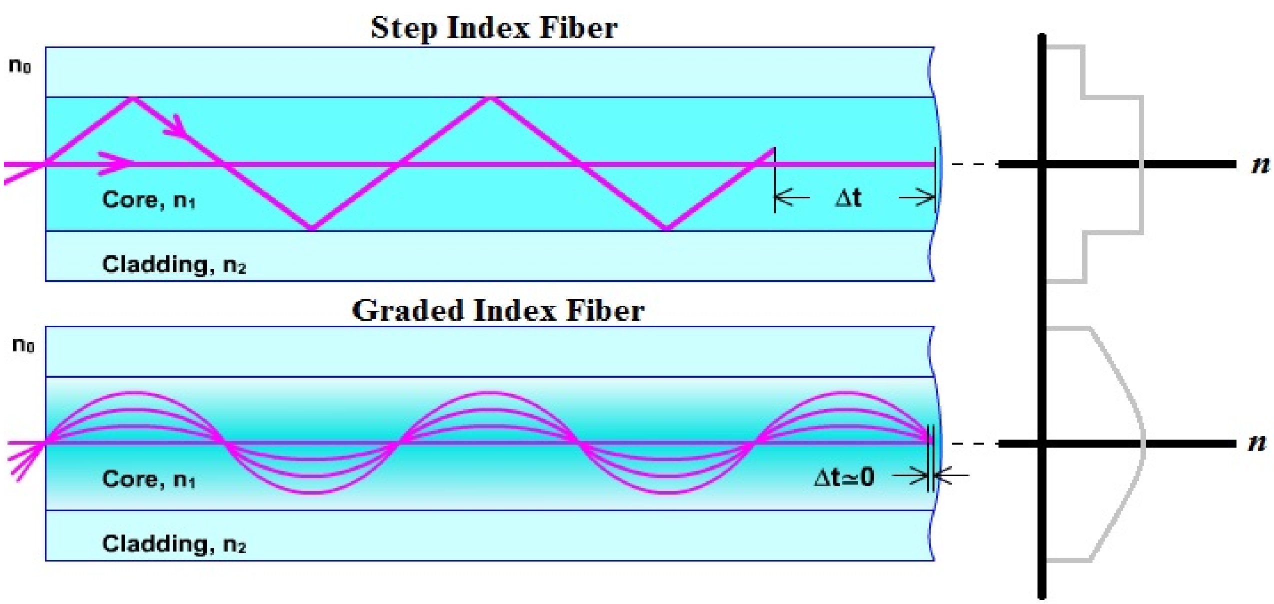
Thus

$$\sin\theta_a = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

Numerical Aperture (NA) is defined as

$$NA = n_a \sin\theta_a = n_1 \sin\theta_1 = n_1\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}$$

Step-Index Fiber and Graded-Index Fiber



Step-Index Fiber

What is Step-Index Fiber : The refractive index distribution like a *step function* , with a sudden change in refractive index between the core-cladding boundary.

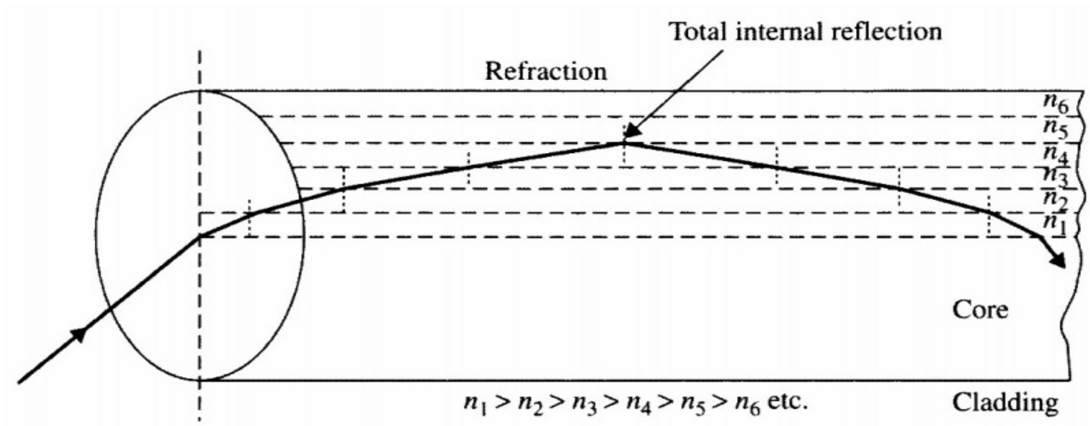
$$n = \begin{cases} n_1 & \text{core} & r \leq a \\ n_2 & \text{cladding} & a < r \leq d \\ n_0 & \text{air} & r > d \end{cases}$$

Graded Index Fiber

What is graded-index fiber : the refractive index distribution is *continuous* , it gradually changing from the center of the core to the code-cladding boundary

$$n(r) = \begin{cases} n_1\sqrt{1 - 2\Delta\left(\frac{r}{a}\right)^\alpha} & \text{core} & r < a \\ n_1\sqrt{1-2\Delta} & \text{cladding} & d > r \geq a \\ n_0 & \text{air} & r \geq d \end{cases} \qquad \Delta =$$

α is the parabolic index, the best α with least dispersion is 2



Because the refractive index in graded-index fiber is changing, thus the light wave *bend* along the core to produce a *sin/cos* locus.

Pulse Broadening and Cross talk

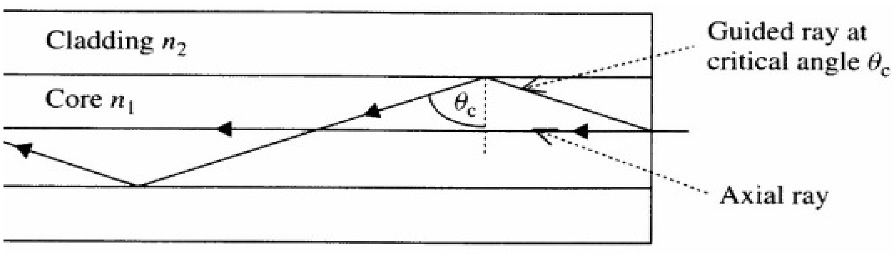
Because of the different propagation time for different mode of wave in the fiber , there will be *pulse broadening*

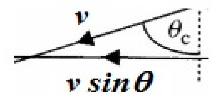
The negative effect of such broadening is the creation of *cross talk* , 2 originally separated wave overlap with each other and cause error.

Intermodal (Multi-mode) Dispersion

What is intermodal dispersion
The pulse broadening of different mode of waves is intermodal dispersion.

The intermodal dispersion of step-index fiber





The fastest wave is the wave along the center (θ tp the normal is 90^o)

$$t_{min} = \frac{L}{v} = \frac{L}{c/n} = \frac{L}{c}n_1$$

The slowest wave is the wave incident with the critical angle (there will be no total internal reflection if $\theta < \theta_c$)

$$t_{max} = \frac{L}{v\sin\theta_c} = \frac{L}{\frac{c}{n_1}\frac{n_2}{n_1}} = \frac{L}{c}\frac{n_1^2}{n_2}$$

Thus the pulse spread in time

$$\delta t = t_{Max} - t_{min} = \frac{L}{c}\left(\frac{n_1^2}{n_2} - n_1\right) = \frac{L}{c}\frac{n_1^2}{n_2}\left(\frac{n_1 - n_2}{n_1}\right) = \frac{L}{c}\frac{n_1^2}{n_2}\Delta$$

$\Delta = \dfrac{n_1 - n_2}{n_1}$

Therefore the *intermodal dispersion*

$$D_{intermodal} = \frac{1}{c}\frac{n_1^2}{n_2}\Delta$$

The intermodal dispersion of graded-index fiber
Using ray optics, the delay between fastest and slowest mode

$$\delta t = \frac{Ln_1\Delta^2}{2c}$$

Using EM theory, the delay between fastest and slowest mode

$$\delta t = \frac{Ln_1\Delta^2}{8c}$$

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