Waveguide Dispersion

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We first consider the normalized propagation constant, and then the group velocity, and thus the group delay and finally the spectral spread

For a complex propagation constant $k = \gamma = \alpha + j\beta$, where β is the propagation constant, α is the antennauation constant

Consider the ratio $\frac{\beta}{|\gamma|} = \frac{\beta}{k}$
Then the normalized propagation constant to n_{clad} is thus

$$b = \frac{\frac{\beta}{k} - n_{clad}}{n_{core} - n_{clad}}$$

Express β in terms of k

$$\beta = kb\left(n_{core} - n_{clad}\right) + kn_{clad}$$

Therefore the group velocity and unit group delay

$$v_{wg} = \frac{d\omega}{d\beta} = c\frac{dk}{d\beta} \qquad \tau_w = \frac{1}{v_{wg}} = \frac{1}{c}\frac{d\beta}{dk}$$

To find $\frac{d\beta}{dk}$

$$\frac{d\beta}{dk} = \frac{d}{dk}\left[kb\left(n_{core} - n_{clad}\right) + kn_{clad}\right]$$

$$\frac{d\beta}{dk} = n_{clad} + \left(n_{core} - n_{clad}\right)\frac{d\left(bk\right)}{dk}$$

Since

$$V = \sqrt{n_{core}^2 - n_{clad}^2}ka$$

Thus

$$\frac{d\left(bk\right)}{dk} = \frac{d(bk)\sqrt{n_{core}^2 - n_{clad}^2}a}{dk\sqrt{n_{core}^2 - n_{clad}^2}a} = \frac{d\left(bV\right)}{dV}$$

Thus

$$\frac{d\beta}{dk} = n_{clad} + \left(n_{core} - n_{clad}\right)\frac{d\left(bV\right)}{dV}$$

And therefore the group delay

$$\tau_{wg} = L\tau_w = \frac{L}{c}\frac{d\beta}{dk}$$

$$\tau_{wg} = \frac{L}{c}\left[n_{clad} + \left(n_{core} - n_{clad}\right)\frac{d\left(bV\right)}{dV}\right]$$

$$= \frac{L}{c} n_{clad} \Big[1 + \frac{n_{core} - n_{clad}}{n_{clad}} \frac{d\,(bV)}{dV} \Big]$$

$$= \frac{L}{c} n_{clad} \Big[1 + \Delta \frac{d\,(bV)}{dV} \Big]$$

Where $\Delta = \dfrac{n_{core} - n_{clad}}{n_{clad}}$
The waveguide dispersion is thus

$$D_w = \frac{1}{L} \frac{d\tau_{wg}}{dV} \frac{dV}{d\lambda}$$

$$= \frac{1}{L} \frac{d}{dV} \left[\frac{L}{c} n_{clad} \Big[1 + \Delta \frac{d\,(bV)}{dV} \Big] \right] \frac{dV}{d\lambda}$$

$$= \frac{n_{clad}\Delta}{c} \frac{d^2\,(bV)}{dV^2} \Big] \frac{dV}{d\lambda}$$

For $\dfrac{dV}{d\lambda}$

$$V = \sqrt{n_{core}^2 - n_{clad}^2} ka$$

$$V = \frac{2\pi}{\lambda} \sqrt{n_{core}^2 - n_{clad}^2} a$$

$$\frac{dV}{d\lambda} = \frac{-V}{\lambda}$$

Thus

$$D_{wg} = -\frac{n_{clad}\Delta}{c} \frac{d^2\,(bV)}{dV^2} \frac{V}{\lambda}$$

$$-END-$$