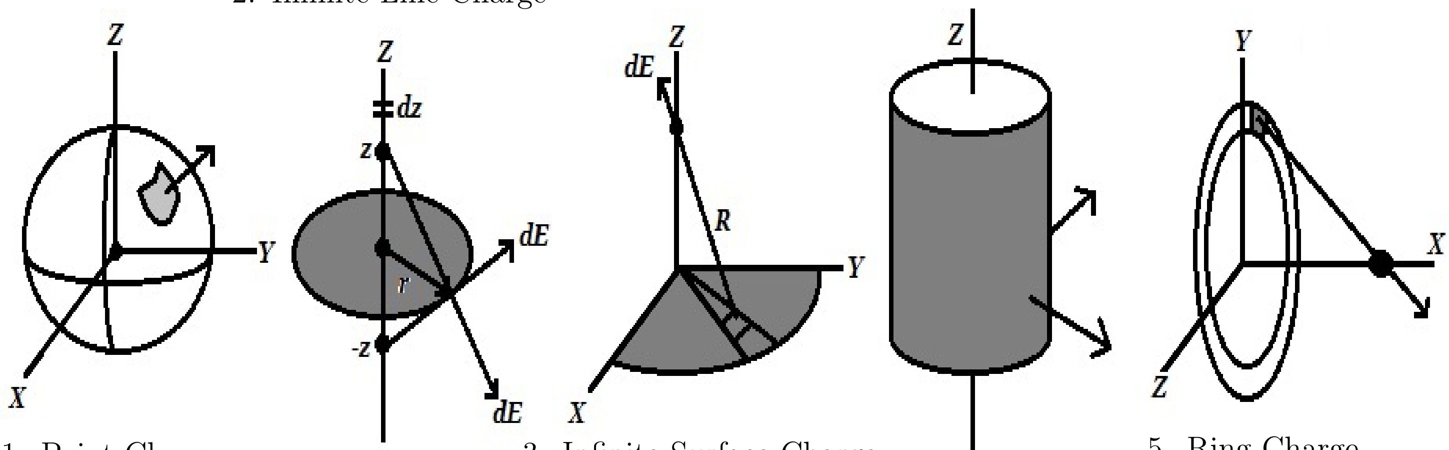
Standard Charge Configurations

May 23, 2013

2. Infinite Line Charge

4. Cylindrical Surface Charge



1. Point Charge

3. Infinite Surface Charge

5. Ring Charge

1. Point Charge (Left most)

Using the Integral form Gauss's Law

$$\oiint \bar{D} \cdot dS = Q_{encl}$$

Thus

$$\varepsilon \bar{E} \left(\int_{\theta=0}^{\theta=\pi} \int_{\phi=0}^{\phi=2\pi} r^2 \sin\theta d\theta d\phi \right) = Q$$

$$\bar{E} = \frac{Q}{\varepsilon r^2 \int_{\theta=0}^{\theta=\pi} \sin\theta d\theta \int_{\phi=0}^{\phi=2\pi} d\phi}$$

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{r}$$

2. Line Charge

Since $\bar{E} = \frac{Q}{4\pi\varepsilon R^2}\hat{R}$, thus $d\bar{E} = \frac{dQ}{4\pi\varepsilon R^2}\hat{R}$

Where

$$R = \sqrt{r^2 + z^2} \;\; \hat{R} = \frac{r\hat{r} - z\hat{z}}{\sqrt{r^2 + z^2}}$$

Thus

$$d\bar{E} = \frac{dQ}{4\pi\varepsilon\left(r^2 + z^2\right)}\frac{r\hat{r} - z\hat{z}}{\sqrt{r^2 + z^2}}$$

Since charge distributed evenly along the line, so

$$dQ = \rho dz$$

Therefore

$$d\bar{E} = \frac{\rho\left(r\hat{r} - z\hat{z}\right)}{4\pi\varepsilon\left(r^2 + z^2\right)^{3/2}}dz$$

Looking at the diagram, the for every charge dQ at $+z$, there is always another charge dQ at $-z$, and the z-component is cancelled, thus

$$d\bar{E} = \frac{\rho r\hat{r}}{4\pi\varepsilon\left(r^2 + z^2\right)^{3/2}}dz$$

$$E = \int_{-\infty}^{\infty} dE$$

$$E = \int_{-\infty}^{\infty} \frac{\rho r\hat{r}}{4\pi\varepsilon\left(r^2 + z^2\right)^{3/2}}dz$$

$$\bar{E} = \frac{\rho}{4\pi\varepsilon}\left[\frac{z}{r^2\sqrt{r^2 + z^2}}\right]_{-\infty}^{\infty}\hat{r}$$

$$\bar{E} = \frac{\rho}{2\pi\varepsilon r}\hat{r}$$

3. Surface Charge

Same as before , since $\bar{E} = \frac{Q}{4\pi\varepsilon R^2}\hat{R}$, thus $d\bar{E} = \frac{dQ}{4\pi\varepsilon R^2}\hat{R}$

Where

$$R = \sqrt{r^2 + z^2} \;\; \hat{R} = \frac{-r\hat{r} + z\hat{z}}{\sqrt{r^2 + z^2}}$$

And the charge is distributed evenly over infinite plane

$$dQ = \rho r dr d\phi$$

$$d\bar{E} = \frac{\rho r dr d\phi}{4\pi\varepsilon\left(r^2 + z^2\right)^{3/2}}\left(-r\hat{r} + z\hat{z}\right)$$

Symmetry : For any charge at $(r, \phi, 0)$, there are 3 other charges in $(r, \phi + \frac{\pi}{2}, 0)$, $(r, \phi + \pi, 0)$, $(r, \phi - \frac{\pi}{2}, 0)$ that the $\hat{r}$ direction field is cancelled.

$$d\bar{E} = \frac{\rho r z dr d\phi}{4\pi\varepsilon\left(r^2+z^2\right)^{3/2}}\hat{z}$$

Therefore

$$E = \int_{\phi=0}^{\phi=2\pi}\int_0^\infty \frac{\rho r z dr d\phi}{4\pi\varepsilon\left(r^2+z^2\right)^{3/2}}\hat{z} = \frac{\rho}{2\varepsilon}\hat{z}$$

4. Cylindrical Surface Charge

Apply Gauss's Law

$$Q_{encl} = \oiint_S \bar{D}\cdot d\bar{S}$$

$$\rho L = Q_{encl} = \bar{D}\oiint_S d\bar{S} = \bar{D}2\pi r L$$

Thus

$$\bar{D} = \frac{\rho}{2\pi r}\hat{r} \qquad \bar{E} = \frac{\rho}{2\pi\varepsilon r}\hat{r}$$

5. Ring Charge

$d\bar{E} = \dfrac{dQ}{4\pi\varepsilon R^2}\hat{R}$, where

$$dQ = \rho ds = \left(\frac{Q}{2\pi r}\right)(r d\theta) = \frac{Q}{2\pi}d\theta$$

$$R = \sqrt{r^2+x^2} \qquad \hat{R} = \frac{-r\hat{r}+x\hat{x}}{\sqrt{r^2+z^2}}$$

Because of symmetry, $r-$term cancelled

$$d\bar{E} = \frac{1}{4\pi\varepsilon\left(r^2+x^2\right)}\frac{Q}{2\pi}d\theta\frac{x\hat{x}}{\sqrt{r^2+x^2}} = \frac{Qx\hat{x}d\theta}{8\pi^2\varepsilon\left(r^2+x^2\right)^{3/2}}$$

Therefore

$$E = \int_{\theta=0}^{\theta=2\pi}\frac{Qx\hat{x}d\theta}{8\pi^2\varepsilon\left(r^2+x^2\right)^{3/2}}$$

$$E = \frac{Qx\hat{x}}{8\pi^2\varepsilon\left(r^2+x^2\right)^{3/2}}\int_{\theta=0}^{\theta=2\pi}d\theta = \frac{Qx}{4\pi\varepsilon\left(r^2+x^2\right)^{3/2}}\hat{x}$$

Approximation : When $x \gg r$ (a point much much far away from the origin)

$$\left(r^2+x^2\right)^{3/2} \approx x^3$$

$$E \approx \frac{Q}{4\pi\varepsilon x}\hat{x} \qquad \text{(Appear like a point charge !)}$$

$-END-$