PN Junction

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Reference Sedra / Smith , *Microelectronic Circuits*

1 Maxwell's Eqautions Review : Poisson's Equation for PNJ

1.1 Gauss Law for E field

The total enclosed charge $Q_{encl.}$ insde closed surface S is given by $Q_{encl} \propto \Phi_{E,S}$

i.e.

$$Q_{encl} \propto \oiint_S \mathbf{E} \cdot d\mathbf{S}$$

With the proportional constant

$$Q_{Encl} = \epsilon \oiint_S \mathbf{E} \cdot d\mathbf{S}$$

Charge Density

$$\rho = \frac{dQ}{dV} \qquad \sigma = \frac{dQ}{dS} \qquad \lambda = \frac{dQ}{dl}$$

1.2 Poisson's Equation

$$\iiint_V \rho dV = Q_{encl.} = \epsilon \oiint_S \mathbf{E} \cdot d\mathbf{S}$$

Recall, the Gauss's Divergence Theorem

$$\oiint \mathbf{A} \cdot d\mathbf{S} = \iiint \nabla \cdot \mathbf{A} dV$$

Thus

$$\iiint_V \rho dV = \epsilon \iiint_V \nabla \cdot \mathbf{E} dV$$

Equalize the integrand

$$\rho = \epsilon \nabla \cdot \mathbf{E}$$

Rearrange

$$\frac{\rho}{\epsilon} = \nabla \cdot \mathbf{E}$$

Recall, the relation of E and V

$$\mathbf{E} = -\nabla V$$

Combine the equations

$$\nabla^2 V = -\frac{\rho}{\epsilon} \qquad (Poisson's\ Eqaution)$$

1.3 Poisson's Equation for PNJ

Simplify to one dimensional case

$$\frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\epsilon}$$

With carrier concentration in for PN-Junction

$$\frac{\partial^2 V}{\partial x^2} = -\frac{q}{\epsilon}\rho'$$

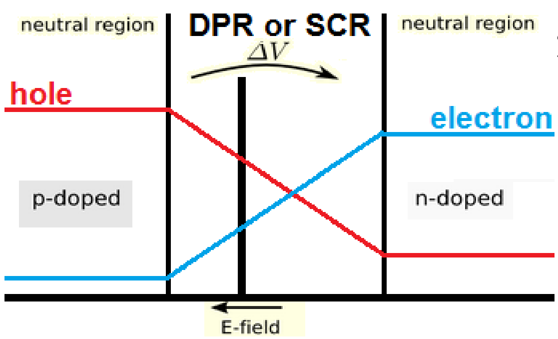
PN-Junction carrier concentration (Affected by dopping)

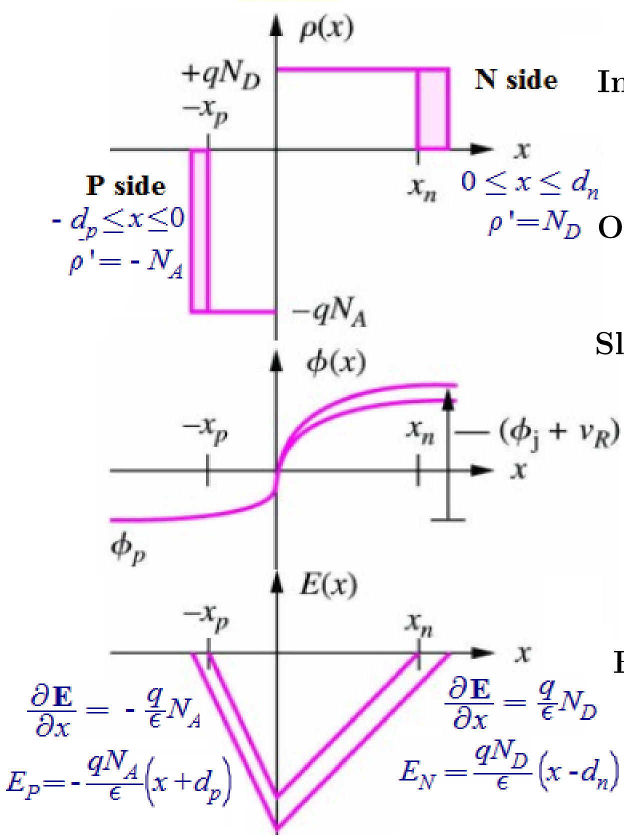
$$\rho' = p - n + N_D - N_A$$

With assumption that donor, acceptor 100% ionized
The final Poisson's Equation for PN-Junction

$$\frac{\partial^2 V}{\partial x^2} = -\frac{q}{\epsilon}\left[p - n + N_D - N_A\right]$$

2 Analysis of the Depletion Region of PN-Junction





2.1 Approximations

- $n\,,\,p\;\ll N_A\,,\,N_D$: Depletion region is *depleted* of carriers.
- No net charge outside depletion region : Q-neutral

Thus in the P side : $(\ -d_p \le x \le 0\ )$

$$N_A \gg n_p, p_p \quad \rho' = -N_A$$

In the N side : $(0 \le x \le d_n\ )$

$$N_D \gg n_n, p_n \quad \rho' = N_D$$

Outside : $(\ x > d_n\ ,\ x < -d_p\ )$

$$\rho' = 0$$

Slope of E-x diagram

$$\begin{cases} \dfrac{\partial \mathbf{E}}{\partial x} = \dfrac{q}{\epsilon}N_D & \text{N-side} \\[2ex] \dfrac{\partial \mathbf{E}}{\partial x} = -\dfrac{q}{\epsilon}N_A & \text{P-side} \end{cases}$$

* E-field continuous at $x = -d_p,\ 0\ ,\ d_n$

Build-in E-field

$\exists$dopant ions in depletion region

$\Rightarrow E - field \Rightarrow$ Stop further carrier flow

2.2 Build-in Potential

In equilibrium , carrier stop flowing across.

Fermi–Dirac statistics

$$f(E) = \frac{1}{e^{(E-E_F)/kT} - 1}$$

Electrons in conduction band

$$n = N_C f(E_C) = N_C \frac{1}{e^{(E_C-E_F)/kT} - 1}$$

As E_C is few level higher than E_F , so apply approximation $e^x - 1 \cong e^x$

$$n \cong N_C e^{-(E_C-E_F)/kT} \propto e^{-(E_C-E_F)/kT}$$

Remark. E_C, E_V , E_F are energy of the specific energy bands, not E-field

At equilibrium :

$$n_p \cong N_C e^{-(E_{CP}-E_F)/kT} \qquad n_n \cong N_C e^{-(E_{CN}-E_F)/kT}$$

$$n_p = \frac{n_i^2}{N_A} \text{ (Neutral P-side)} \qquad n_n = N_D \text{ (Netural N-side)}$$

$$\frac{N_D N_A}{n_i^2} = \frac{n_n}{n_p} = e^{(E_{CP}-E_{CN})/kT} = e^{qV_{bi}/kT}$$

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

Recall, the $Thermal\ Voltage$

$$V_T = \frac{kT}{q}$$

Build-in voltage of PN-Junction

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = V_T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

2.3 E-field in PNJ

$$\frac{\partial \mathbf{E}}{\partial x} = \frac{q}{\epsilon}\rho'$$

$$\begin{cases} \dfrac{\partial \mathbf{E}}{\partial x} = \dfrac{q}{\epsilon} N_D & 0 \le x \le d_n \quad N-side \\ \dfrac{\partial \mathbf{E}}{\partial x} = -\dfrac{q}{\epsilon} N_A & -d_p \le x \le 0 \quad P-side \end{cases} \quad - \int \to \quad \begin{cases} E_N = \dfrac{qN_D}{\epsilon} x + C_1 & N-side \\ E_P = -\dfrac{qN_A}{\epsilon} x + C_2 & P-side \end{cases}$$

Apply *Boundary Condition*

$$\begin{cases} E_P = 0 & x = -d_p \\ E_N = 0 & x = d_n \end{cases} \Rightarrow \begin{cases} C_1 = -\dfrac{qN_D}{\epsilon} d_n \\ C_2 = -\dfrac{qN_A}{\epsilon} d_p \end{cases}$$

Thus

$$\begin{cases} E_N = \dfrac{qN_D}{\epsilon}(x - d_n) & N - side \\ E_P = -\dfrac{qN_A}{\epsilon}(x + d_p) & P - side \end{cases}$$

Max. E-field

$$E_j = E_p(x = 0) = E_n(x = 0) = -\frac{qN_A d_p}{\epsilon} = -\frac{qN_D d_n}{\epsilon}$$

$$\iff N_A d_p = N_D d_n \qquad (Q - neutrality)$$

2.4 Build-in Potential Barrier

$$V = -\int E dx$$

$$\begin{cases} E_P = -\dfrac{qN_A}{\epsilon}(x + d_p) & N - side \\ E_N = \dfrac{qN_D}{\epsilon}(x - d_n) & P - side \end{cases}$$

$$\begin{cases} V_P = -\int E_p dx = \int \dfrac{qN_A}{\epsilon}(x + d_p) dx = \dfrac{qN_A}{\epsilon} \int (x + d_p) dx \\ V_N = -\int E_n dx = -\int \dfrac{qN_D}{\epsilon}(x - d_n) dx = -\dfrac{qN_D}{\epsilon} \int (x - d_n) dx \end{cases}$$

$$\begin{cases} V_P = \dfrac{qN_A}{\epsilon}\left[\dfrac{x^2}{2} + d_p x\right] & C = 0 \text{ By} V(0) = 0 \\ V_N = -\dfrac{qN_D}{\epsilon}\left[\dfrac{x^2}{2} - d_n x\right] & C = 0 \text{ By } V(0) = 0 \end{cases} \qquad \begin{cases} V_P(-d_p) = -\dfrac{qN_A}{2\epsilon} d_p^2 \\ V_N(d_n) = \dfrac{qN_D}{2\epsilon} d_n^2 \end{cases}$$

Build-in Potential Barrier

$$V_{np} = V_N(d_n) - V_P(-d_P) = \frac{q}{2\epsilon}\left(N_D d_n^2 + N_A d_p^2\right)$$

2.5 Width of Depletion Region

Apply Q-neutrality

$$N_A d_p = N_D d_n \qquad \frac{N_A}{N_D} = \frac{d_n}{d_p}$$

Into Build-in Potential Barrier

$$V_{np} = \frac{q}{2\epsilon}\left(N_D d_n^2 + N_A d_p^2\right) \Rightarrow \begin{cases} V_{np} = \frac{q}{2\epsilon}\left(N_D\left[\frac{N_A}{N_D}d_p\right]^2 + N_A d_p^2\right) = \frac{q}{2\epsilon}\left(\frac{N_A+N_D}{N_D}\right)N_A d_p^2 \\ V_{np} = \frac{q}{2\epsilon}\left(N_D d_n^2 + N_A\left[\frac{N_D}{N_A}d_n\right]^2\right) = \frac{q}{2\epsilon}\left(\frac{N_A+N_D}{N_A}\right)N_D d_n^2 \end{cases}$$

$$\begin{cases} d_p = \sqrt{\frac{2\epsilon V_{np}}{q}\left(\frac{N_D}{N_A+N_D}\right)\frac{1}{N_A}} = \frac{1}{N_A}\sqrt{\frac{2\epsilon V_{np}}{q}N_j} \ or \ \frac{N_j}{N_A}\sqrt{\frac{2\epsilon V_{np}}{qN_j}} \\ d_n = \sqrt{\frac{2\epsilon V_{np}}{q}\left(\frac{N_A}{N_A+N_D}\right)\frac{1}{N_D}} = \frac{1}{N_D}\sqrt{\frac{2\epsilon V_{np}}{q}N_j} \ or \ \frac{N_j}{N_D}\sqrt{\frac{2\epsilon V_{np}}{qN_j}} \end{cases}$$

Where $N_j = N_A||N_D = \frac{N_A N_D}{N_A+N_D}$, $\frac{N_j}{N_A} = \frac{N_D}{N_A+N_D}$, $\frac{N_j}{N_D} = \frac{N_A}{N_A+N_D}$

Thus, the width

$$w = d_n + d_p = \sqrt{\frac{2\epsilon V_{np}}{qN_j}}\underbrace{\left\{\frac{N_j}{N_A}+\frac{N_j}{N_D}\right\}}_{1} = \sqrt{\frac{2\epsilon V_{np}}{qN_j}} = \sqrt{\frac{2\epsilon V_{np}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

2.6 Junction Capacitance

$$Q = qN_A d_p \quad (= qN_D d_n)$$

$$Q = qN_j\sqrt{\frac{2\epsilon V_{np}}{qN_j}} = \sqrt{2\epsilon q N_j V_{np}}$$

With external voltage source V_R, the PN-Junction barrier voltage will become : $V_{np} \xrightarrow{V_R} V_{np} + V_R$

$$Q = \sqrt{2\epsilon q N_j\left(v_{np}+V_R\right)} = \sqrt{2\epsilon q N_j}\cdot\sqrt{v_{np}+V_R}$$

$$C_j = \frac{dQ}{dV_R}|_{V_R \text{very small}} = \sqrt{2\epsilon q N_j}\cdot\frac{1}{2\sqrt{v_{np}+V_R}} = \sqrt{\frac{\epsilon q N_j}{2\left(v_{np}+V_R\right)}}$$

For C_{j0}, with no applied voltage

$$C_{j0} = \sqrt{\frac{\epsilon q}{2}\left(\frac{N_A N_D}{N_A+N_D}\right)\frac{1}{v_{np}}}$$

Therefore

$$C_j = \sqrt{\frac{\epsilon q N_j}{2\left(v_{np}+V_R\right)}} = \sqrt{\frac{\epsilon q N_j}{2\left(v_{np}+V_R\right)}\frac{v_{np}}{v_{np}}} = \sqrt{\frac{\epsilon q N_j}{2v_{np}}}\cdot\sqrt{\frac{v_{np}}{v_{np}+V_R}} = \frac{C_{j0}}{\sqrt{1+\frac{v_R}{v_{np}}}}$$

3 Summary

Name / Description	Equation
General Poisson's Equation	$\nabla^2 V = -\frac{\rho}{\epsilon}$
1D Poisson's Eqaution for PN-Junction	$\frac{\partial^2 V}{\partial x^2} = -\frac{q}{\epsilon}\left[p - n + N_D - N_A\right]$
Carrier Concentrations in PNJ	$\rho' = \begin{cases} 0 & x < -d_p \\ -N_A & -d_p \leq x \leq 0 \\ N_D & 0 \leq x \leq d_n \\ 0 & x > d_n \end{cases}$
Fermi-Dirac Statistic	$f\left(E\right) = \frac{1}{e^{(E-E_F)/kT} - 1}$
Intrinsic Carrier Concentration	$n_i^2 = B'T^3 e^{-\frac{E_g}{kT}}$
Thermal Voltage	$V_T = \frac{kT}{q}$
e- in depletion region	$n_n = N_C e^{-(E_{CN}-E_F)/kT}$ N-side SCR $n_p = N_C e^{-(E_{CP}-E_F)/kT}$ P-side SCR
$e-$ in neutral rehion	$n_n = N_D$ $n_p = \frac{n_i^2}{N_A}$
Build-in Voltage across SCR	$V_{bi} = \frac{E_{CP} - E_{CN}}{q} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$
E-field in PNJ	$E_N = \frac{qN_D}{\epsilon}\left(x - d_n\right)$ N-side $E_P = -\frac{qN_A}{\epsilon}\left(x + d_p\right)$ P-side $E_j = E_N(0) = E_P(0) = -\frac{qN_D}{\epsilon}d_n = \frac{qN_A}{\epsilon}d_p$ Max
Charge Neutrality	$\frac{N_A}{N_D} = \frac{d_n}{d_p}$
Build-in Potential Barrier	$V = -\int E dx = \begin{cases} V_P = \frac{qN_A}{\epsilon}\left[\frac{x^2}{2} + d_p x\right] & \text{P-side} \\ V_N = -\frac{qN_D}{\epsilon}\left[\frac{x^2}{2} - d_n x\right] & \text{N-side} \end{cases}$
SCR distance	$d_p = \frac{N_j}{N_A}\sqrt{\frac{2\epsilon V_{np}}{qN_j}} \qquad d_n = \frac{N_j}{N_D}\sqrt{\frac{2\epsilon V_{np}}{qN_j}}$
N_j	$N_j = N_A \vert\vert N_D$
Space Charge Region Width	$w_0 = d_p + d_n = \sqrt{\frac{2\epsilon}{q}V_0\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$
SCR Width with external source	$w = w_0\sqrt{1 + \frac{v_R}{v_j}}$
PNJ Capacitance	$C_j = \sqrt{\frac{\epsilon q N_j}{2\left(v_{np} + V_R\right)}}$

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