Some Standard Electric Circuits

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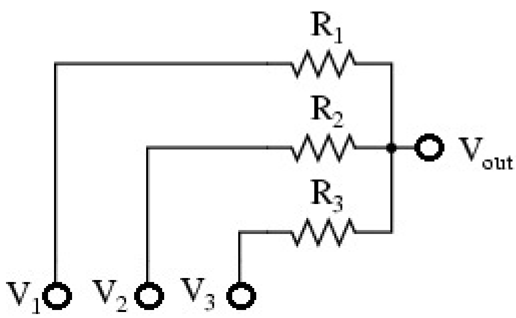
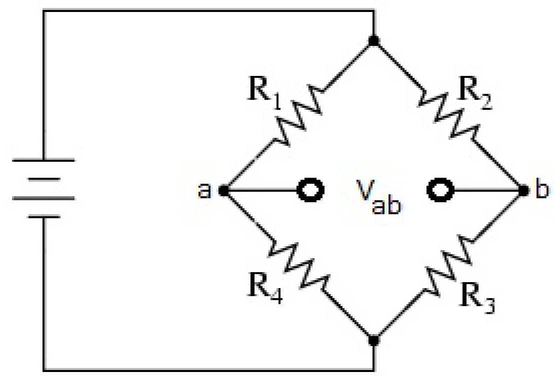
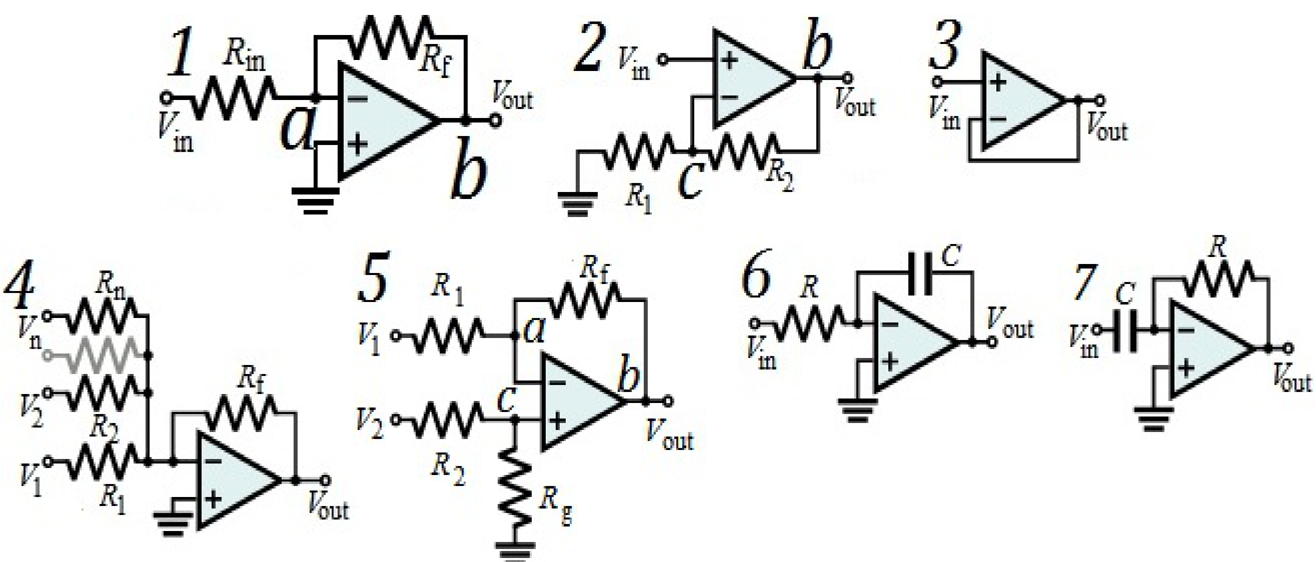
Some Resistor Network





Some OpAmp Circuits



The Virtual Short Concept for $\pm$ pins in an OpAmp

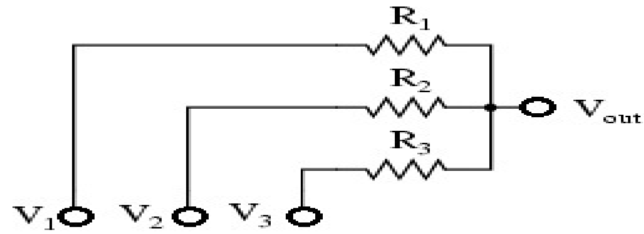
The pin $-$ and $+$ are virtual shorted : $V_- = V_+$

(This is because of finite output and infinite gain of ideal OpAmp $v_{out} = A(v_+ - v_-)$)

Thus, when one of pin is grounded, then both pins are grounded.

1 The Resistor Network

1.1 Averager



The current of R_1 is $\dfrac{V_1}{R_1}$

Thus at the output point, all the current sum together : $I_{out} = \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}$

Since the 3 resistor is in parallel, so the equivalent resistance is $\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right)^{-1} = \dfrac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$

Thus the output voltage is $I_{out}R_{eq} = \left(\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right)\left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right)^{-1} = \dfrac{\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

When resistor are equal

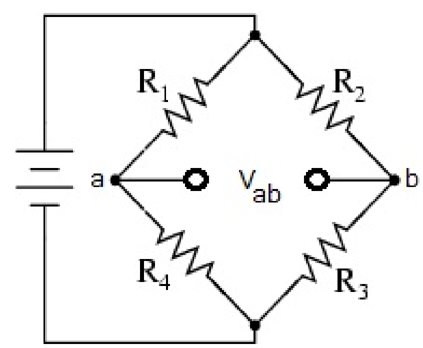
$$R_{eq} = \frac{R}{3} \qquad I_{out} = \frac{V_1 + V_2 + V_3}{R} \qquad V_{out} = \frac{V_1 + V_2 + V_3}{3}$$

Generalized, for n-resistor averager The equivalent resistance is

$$R_{eq}(n) = \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}\right)^{-1} = \frac{R_1R_2...R_n}{R_1R_2...R_{n-1} + R_1R_2...R_{n-2}R_n + ... + R_2R_3...R_n}$$

$$R_{eq}(n) = \left(\sum_{i=1}^{n} \frac{1}{R_i}\right)^{-1} = \frac{\prod_{i=1}^{n} R_i}{\sum_{i=1}^{n} \prod_{\substack{j=1 \\ j \neq i}}^{n} R_i} = \frac{\text{Sum of } R \text{ taken } n \text{ at a time}}{\text{Sum of } R \text{ taken } n-1\text{at a time}}$$

(Analogy to Algebra of Polynomial)



The output current is $I_{out} = \dfrac{V_1}{R_1} + ... + \dfrac{V_n}{R_n}$

And the ouput voltage is $V_{out} = \dfrac{\dfrac{V_1}{R_1} + ... + \dfrac{V_n}{R_n}}{\dfrac{1}{R_1} + ... + \dfrac{1}{R_n}}$

1.2 Birdge

The path R_1R_4 is parallel to path R_2R_3 , thus voltage across $R_1 + R_4$ is equal to $R_2 + R_3$

By potential divider, $V_a = V_{in}\dfrac{R_4}{R_1 + R_4}$, $V_b = V_{in}\dfrac{R_3}{R_2 + R_3}$

Thus, $V_{ab} = V_a - V_b = V_{in}\left(\dfrac{R_4}{R_1 + R_4} - \dfrac{R_3}{R_2 + R_3}\right)$

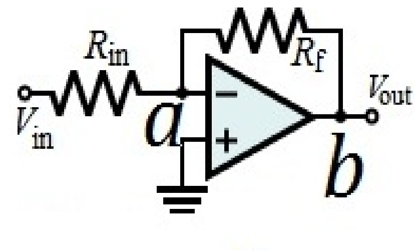
When $V_{ab} = 0$, then $V_a = V_b$, and thus

$$\frac{R_4}{R_1 + R_4} = \frac{R_3}{R_2 + R_3} \quad \Rightarrow \frac{R_4}{R_1 + R_4}\frac{1/R_4}{1/R_4} = \frac{R_3}{R_2 + R_3}\frac{1/R_3}{1/R_3} \quad \Rightarrow \frac{1}{\dfrac{R_1}{R_4} + 1} = \frac{1}{\dfrac{R_2}{R_3} + 1} \Rightarrow \frac{R_2}{R_3} + 1 = \frac{R_1}{R_4} + 1$$

$$\Longrightarrow \frac{R_1}{R_4} = \frac{R_2}{R_3}$$

2 OpAmp Circuits

2.1 Inverting Amplifier



Input current is $I_{in} = \dfrac{V_{in}}{R_{in}}$

Since the ±pin is virtual grounded ($V_a = 0$), so the all current go into R_f

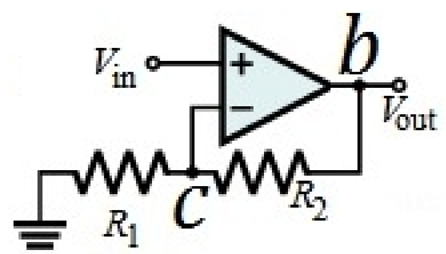
Thus the voltage of R_f is $V_{R_f} = I_{in}R_f = V_{in}\dfrac{R_f}{R_{in}}$

By KVL : $V_a = V_{R_f} + V_b \iff 0 = V_{in}\dfrac{R_f}{R_{in}} + V_{out}$

Thus, the gain is $\dfrac{V_{out}}{V_{in}} = -\dfrac{R_f}{R_{in}}$

When $R_f = R_{in}$, the OpAmp become a phase invertor , $V_{out} = -V_{in}$

2.2 Non-inverting Amplifier



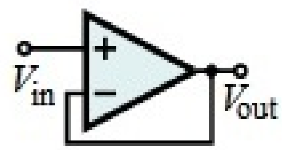
Input voltage V_{in}

By potential divider at point C : $V_C = V_{out}\dfrac{R_1}{R_1 + R_2}$

By virtual short of ±pin, $V_C = V_{in}$

So the gain is $\dfrac{V_{out}}{V_{in}} = \dfrac{R_1 + R_2}{R_1} = 1 + \dfrac{R_2}{R_1}$

2.3 Voltage Follower



The input voltage V_{in} equal to V_-

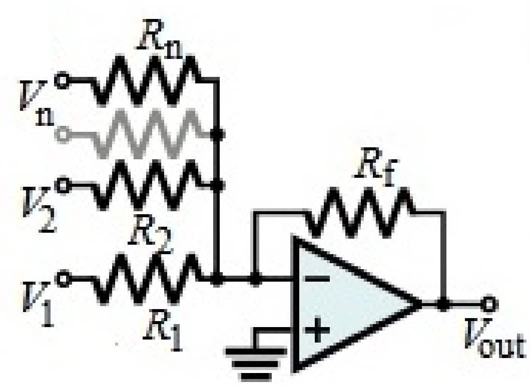
The output voltage V_{out} equal to V_+

By virtual short in ±pin, $V_- = V_+$

Thus $V_{out} = V_{in}$

So the gain is $\dfrac{V_{out}}{V_{in}} = 1$

2.4 Weighted Summer



Notice that the resistor network is an averager.

The input current of resistor k : $I_k = \dfrac{V_{in,k}}{R_k}$

By KCL, the total current in node a will be $I_a = \sum_{all\,k} I_k$

Since + pin is grounded, so by virtual short in ±pin, − pin is also grounded ($V_a = 0$)

all current go into R_f Thus the voltage of R_f is

$$V_{R_f} = I_f R_f = I_a R_f = \left(\sum I_k\right) R_f = \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + ... + \frac{V_n}{R_n}\right) R_f = V_1\frac{R_f}{R_1} + V_2\frac{R_f}{R_2} + ... + V_n\frac{R_f}{R_n}$$

By KVL, $V_a = V_{R_f} + V_b \iff 0 = \left(V_1\dfrac{R_f}{R_1} + V_2\dfrac{R_f}{R_2} + ... + V_n\dfrac{R_f}{R_n}\right) + V_{out}$

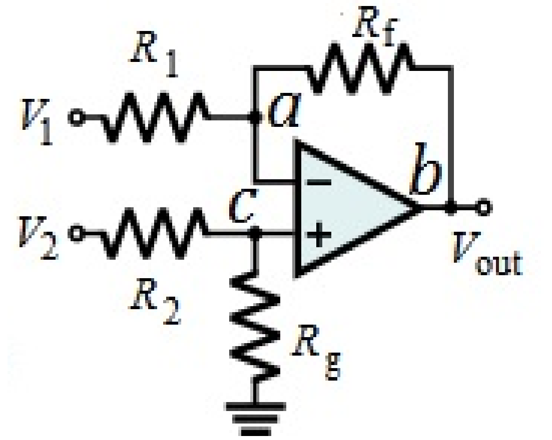
Thus $V_{out} = -\left(V_1\dfrac{R_f}{R_1} + V_2\dfrac{R_f}{R_2} + ... + V_n\dfrac{R_f}{R_n}\right)$

If all the input voltages are equal, the gain is $\dfrac{V_{out}}{V_{in}} = \dfrac{R_f}{R_1} + \dfrac{R_f}{R_2} + ... + \dfrac{R_f}{R_n}$

2.5 Differential Amplifier

Virtual short in ±pins, so $V_a = V_c$, but V_a , $V_c \neq 0$

If $V_1 = 0$, this amplifier become non-inverting amplifier , At point C , $V_c = V_2\frac{R_g}{R_2+R_g}$, this V_c is the V_{in} in non-inverting amplifer



If $V_2 = 0$, this amplifier become inverting amplifier, and

$V_{out1} = -V_1\frac{R_f}{R_1}$

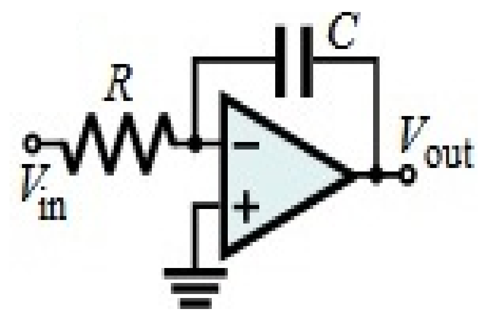
Thus, $V_{out2} = V_c\frac{R_1+R_f}{R_1} = V_2\frac{R_g}{R_2+R_g}\frac{R_1+R_f}{R_1}$

The total output is $V_{out} = V_{out1} + V_{out2}$

$$= -V_1\frac{R_f}{R_1} + V_2\frac{R_g}{R_2+R_g}\frac{R_1+R_f}{R_1}$$

For $R_1 = R_2$, $R_g = R_f$: $V_{out} = (V_2 - V_1)\frac{R_f}{R_1}$, thus, the output is proportional to the difference between 2 inputs

2.6 Integrator



The input current is $I_{in} = \frac{V_{in}}{R}$

Since + pin is GND and virtual short of ±pin,
$V_a = 0$, so all current goes into the capacitor

Capacitor charge-capacitance-voltage relation is : $Q_{cap} = V_{cap}C$

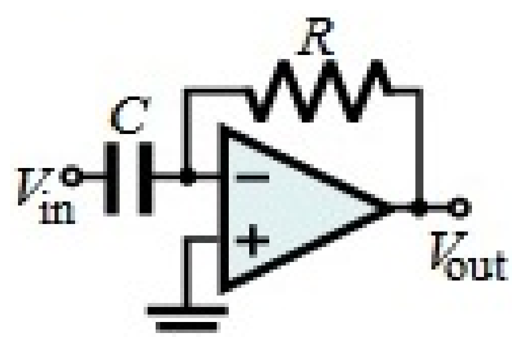
Thus $I_{cap} = \frac{dQ_{cap}}{dt} = C\frac{dV_{cap}}{dt}$

$I_{in} = I_{cap} \iff \frac{V_{in}}{R} = C\frac{dV_{cap}}{dt} \iff V_{cap} = \frac{1}{RC}\int_0^t V_{in}dt$, assume $V_{cap}(t=0) = 0$

By KVL : $V_a = V_{Cap} + V_b \iff 0 = V_{cap} + V_{out} \iff V_{out} = -V_{cap}$

Thus $V_{out} = \frac{-1}{RC}\int_0^t V_{in}dt$

2.7 Differentiator



The capacitor equation : $Q_{cap} = CV_{cap}$

$\Longrightarrow I_{cap} = \frac{dQ_{cap}}{dt} = C\frac{dV_{cap}}{dt} = C\frac{dV_{in}}{dt}$

By +pin GND & ±pin virtual short, $I_R = I_{cap}$

$\Longrightarrow V_R = CR\frac{dV_{in}}{dt}$

By KCL : $V_a = V_R + V_b \iff 0 = RC\frac{dV_{in}}{dt} + V_{out}$

$V_{out} = RC\frac{dV_{in}}{dt}$

Remark. If the input is $A\sin\omega t$, then output is $\omega A\cos\omega t$, thus the ouput will have a larger amplitude for high frequency component. Thus differentiator will also amplify the high frequency noise.

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