EM Wave in two media

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The problem specification : assume normal incidence of an EM wave from media 1 to media 2. Given E_1^+(magnitude of incident E-field), ω , σ , μ , ε for 2 media. Find E_1^- (magnitude of reflected wave) and E_2^+ (magnitude of transmitted wave)

Remark. Since σ,μ,ε are given, so γ , α , β , η can be found, with E and η , H can also be found.

The solving procedures

First, using the media parameters to determine the propagation parameters

$$\gamma_1 = \sqrt{j\omega\mu_1\left(\sigma_1 + j\omega\varepsilon_1\right)} \qquad \gamma_2 = \sqrt{j\omega\mu_2\left(\sigma_2 + j\omega\varepsilon_2\right)}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu_1}{\sigma_1 + j\omega\varepsilon_1}} \qquad \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2 + j\omega\varepsilon_2}}$$

Then write down the wave expressions (in phasor)

$$E_1^+(z) = E_1^+ e^{-\gamma_1 z}\hat{x} \qquad E_1^+(z) = E_1^- e^{\gamma_1 z}\hat{x} \qquad E_2^+(z) = E_2^+ e^{-\gamma_2 z}\hat{x}$$

$$H_1^+(z) = H_1^+ e^{-\gamma_1 z}\hat{y} \qquad H_1^+(z) = H_1^- e^{-\gamma_1 z}(-\hat{y}) \qquad H_2^+(z) = H_2^+ e^{-\gamma_2 z}\hat{y}$$

Incident Wave | Reflected Wave | Transmitted Wave

Using $|\eta| = \dfrac{|E|}{|H|}$, $\eta = |\eta| e^{j\angle\eta}$, express H interms of E to solve for those unknowns.

$$E_1^+(z) = E_1^+ e^{-\gamma_1 z}\hat{x} \qquad E_1^-(z) = E_1^- e^{\gamma_1 z}\hat{x} \qquad E_2^+(z) = E_2^+ e^{-\gamma_2 z}\hat{x}$$

$$H_1^+(z) = \frac{E_1^+}{|\eta_1|} e^{-\gamma_1 z} e^{j\angle\eta_1}\hat{y} \qquad H_1^-(z) = -\frac{E_1^-}{|\eta_1|} e^{\gamma_1 z} e^{j\angle\eta_1}\hat{y} \qquad H_2^+(z) = \frac{E_2^+}{|\eta_2|} e^{-\gamma_2 z} e^{j\angle\eta_2}\hat{y}$$

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Finally apply boundary conditions

$$E_{t1} = E_{t2} \qquad D_{n1} = D_{n2} + \rho \qquad H_{t1} = H_{t2} + J_s \qquad B_{n1} = B_{n2}$$

Since in this case there is no source, so $\rho = J = 0$

$$E_{t1} = E_{t2} \qquad D_{n1} = D_{n2} \qquad H_{t1} = H_{t2} \qquad B_{n1} = B_{n2}$$

Or just simply

$$E_{t1} = E_{t2} \qquad H_{t1} = H_{t2}$$

Apply the boundary condition

$$E_1 \quad (z)|_{z=0} = E_2(z)|_{z=0} \qquad \Longleftrightarrow \qquad E_1^+(0) + E_1^-(0) = E_2^+(0)$$

$$H_1(z)|_{z=0} = H_2(z)|_{z=0} \qquad \Longleftrightarrow \qquad \frac{E_1^+}{|\eta_1|} - \frac{E_1^-}{|\eta_1|} = \frac{E_2^+}{|\eta_2|}$$

i.e.

$$\begin{cases} E_1^+ + E_1^- = E_2^+ \\ \dfrac{E_1^+}{|\eta_1|} - \dfrac{E_1^-}{|\eta_1|} = \dfrac{E_2^+}{|\eta_2|} \end{cases} \quad \text{2 equations, 2 unknown}$$

So the solution is

$$E_1^- = \frac{|\eta_2| - |\eta_1|}{|\eta_1| + |\eta_2|} E_1^+ \qquad E_2^+ = \frac{2|\eta_2| E_1^+}{|\eta_1| + |\eta_2|}$$