BCH Code and RS Code Decoding Process

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1 General Steps for decoding process

1. A received code word r is obtained at the receiver , where r is contaminated by noise n.
2. Form the recived code word polynomial $r(x)$
3. Calculate the syndrome S
4. Obtain the error location polynomial $\sigma(x)$
5. Find the position of error in the received code word by using $\sigma(x)$
6. [Only for RS Code] Find the amount of error
7. Correct the received code word

2 BCH Code Decoding Process

Suppose $r = 11001011$

Step1. Form the $r(x)$
$r(x)$ is the polynomial formed by r
Step1.1 Turning r into a coefficient vector
Step1.2 Perform dot product of the coefficient vector to the power vector of x

i.e. $r(x) = r^T \underline{x} = [11001011] \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \end{bmatrix} = 1 + x^1 + x^4 + x^6 + x^7$

Step2. Form the syndrome $S = S_1 S_2 S_3 S_4$
Syndrome bits S_i can be computed as $S_i = r\left(\alpha^i\right)$
e.g. $S_1 = r\left(\alpha^1\right) = 1 + \alpha^1 + \alpha^4 + \alpha^6 + \alpha^7$, then perform algebaric simiplication based on the table obtained from the Galois Field.

Step3. Form the error location polynomial
The error location polynomial is $\sigma(x) = 1 + \sigma_1 x + \sigma_2 x^2$
where $\sigma_1 = S_1 \qquad \sigma_2 = S_1^2 + \dfrac{S_3}{S_1}$
Therefore, only need to calculate S_1 and S_3.

Step4. Find the error location(s) by $\sigma(x)$

Check the values of $\sigma\left(\alpha^k\right)$ and solve for error location by $\sigma\left(\alpha^k\right)\begin{cases} \neq 0 & \text{No error} \\ = 0 & \text{Error at position} n-k \end{cases}$

Then the error vector can be obtained by putting a "1" in the error location(s) and "0" in all other locations.

Step5. Correct the received code word

$$c = r + e$$

3 RS Code Decoding Process

Suppose $r = \alpha 00 \alpha 00 \alpha^3 \alpha^4$

Step1. Form the $r(x)$

$r(x)$ is the polynomial formed by r

Step1.1 Turning r into a coefficient vector

Step1.2 Perform dot product of the coefficient vector to the power vector of x

i.e. $r(x) = r^T \underline{x} = \left[\alpha 00 \alpha 00 \alpha^3 \alpha^4\right] \begin{bmatrix} 1 \\ x \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \end{bmatrix} = \alpha + \alpha x^3 + \alpha^3 x^6 + \alpha^4 x^7$

Step2. Form the syndrome $S = S_1 S_2 S_3 S_4$

Syndrome bits S_i can be computed as $S_i = r\left(\alpha^i\right)$

e.g. $S_1 = r\left(\alpha^1\right) = \alpha + \alpha^4 + \alpha^9 + \alpha^{11}$, then perform algebaric simiplication based on the table obtained from the Galois Field.

Step3. Form the error location polynomial

The error location polynomial is $\sigma(x) = 1 + \sigma_1 x + \sigma_2 x^2$

where $\sigma_{1,2}$ are obtained by solving $\begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix} \begin{bmatrix} \sigma_2 \\ \sigma_1 \end{bmatrix} = \begin{bmatrix} S_3 \\ S_4 \end{bmatrix}$

Step4. Find the error location(s) by $\sigma(x)$

Check the values of $\sigma\left(\alpha^\beta\right)$ and solve for error location by $\sigma\left(\alpha^\beta\right)\begin{cases} \neq 0 & \text{No error} \\ = 0 & \text{Error at position} n-\beta \end{cases}$

Step5. Find the error value(s)

Since the digit is not binary anymore in RS code, so the amount of error has to be calculated. The amount of error e_i can be found by solving

$$S_1 = e_1 \beta_1 + e_2 \beta_2$$
$$S_2 = e_1 \beta_1^2 + e_2 \beta_2^2$$

where e_1 is the first erorr and e_2 is the second error.

Step6. Correct the received code word

$$c = r + e$$