1 Vector Algebra

1. Commutative addition $\bar{A}+\bar{B}=\bar{B}+\bar{A}$
2. Associative addition $\left(\bar{A}+\bar{B}\right)+\bar{C}=\bar{A}+\left(\bar{B}+\bar{C}\right)$
3. Distributive scalar multiplication $n\left(\bar{A}+\bar{B}\right)=n\bar{A}+n\bar{B}$
4. Negative $(-1)\bar{A}=-\bar{A}$, anti-parallel
5. Magnitude $||\bar{A}||_2=\sqrt{A_1^2+A_2^2+...}$
6. Direction : the unit vector $\hat{A}=\frac{1}{|\bar{A}|}\bar{A}$
7. Magnitude-direction representation : $\bar{A}=|\bar{A}|\hat{A}$
8. Component form : $\bar{A}=(A_1,A_2,A_3)=A_1\hat{x}+A_2\hat{y}+A_3\hat{z}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k}=[A_1\ A_2\ A_3]^T$
9. Rectangular form addition $\bar{A}+\bar{B}=(A_1+B_1,A_2+B_2,A_3+B_3)$
10. Rectangular form scalar multiplication : $n\bar{A}=(nA_1,nA_2,nA_3)$
11. Unit vector of x,y,z : $\hat{x}=(1,0,0)$, $\hat{y}=(0,1,0)$, $\hat{z}=(0,0,1)$
12. Dot product, geometry : $\bar{A}\cdot\bar{B}=AB\cos\theta_{AB}\in$scalar
13. Dot product, component form : $\bar{A}\cdot\bar{B}=A_1B_1+A_2B_2+A_3B_3$
14. $\bar{A}\cdot\bar{A}=A_1^2+A_2^2+A_3^2=A^2\iff|A|=\sqrt{\bar{A}\cdot\bar{A}}$
15. Dot product commutative $\bar{A}\cdot\bar{B}=\bar{B}\cdot\bar{A}$
16. Dot product distribution $\bar{A}\cdot\left(\bar{B}+\bar{C}\right)=\bar{A}\cdot\bar{B}+\bar{A}\cdot\bar{C}$
17. $m\left(\bar{A}\cdot\bar{B}\right)=(m\bar{A})\cdot\bar{B}=\bar{A}\cdot\left(m\bar{B}\right)$
18. Orthogonal $\bar{A}\perp\bar{B}\iff\bar{A}\cdot\bar{B}=0$
19. Unit Vector Dot Product : $\hat{x}\cdot\hat{x}=\hat{y}\cdot\hat{y}=\hat{z}\cdot\hat{z}=1$, $\hat{x}\cdot\hat{y}=\hat{x}\cdot\hat{z}=\hat{y}\cdot\hat{z}=0$
20. Cross product , geometry : $\bar{A}\times\bar{B}=AB\sin\theta_{AB}\hat{n}\in\mathbb{R}^3$ vector , $\hat{n}\perp AB$, $\theta_{AB}\in[0,\pi]$
21. Angle between 2 vectors $\theta=\cos^{-1}\frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}=\sin^{-1}\frac{\left|\bar{A}\times\bar{B}\right|}{|\bar{A}||\bar{B}|}$
22. Cross product skew-commutative : $\bar{A}\times\bar{B}=-\bar{B}\times\bar{A}$
23. $\left|\bar{A}\times\bar{B}\right|=\left|\bar{B}\times\bar{A}\right|$
24. Cross product distributive : $\bar{A}\times\left(\bar{B}+\bar{C}\right)=\bar{A}\times\bar{B}+\bar{A}\times\bar{C}$
25. $m\left(\bar{A}\times\bar{B}\right)=(m\bar{A})\times\bar{B}=\bar{A}\times\left(m\bar{B}\right)$
26. Linear dependent : $\bar{A}=k\bar{B}\iff\bar{A}\times\bar{B}=0$
27. Unit vector cross product : $\hat{x}\times\hat{x}=\hat{y}\times\hat{y}=\hat{z}\times\hat{z}=0$, $\hat{x}\times\hat{y}=\hat{z}$, $\hat{y}\times\hat{z}=\hat{x}$, $\hat{z}\times\hat{x}=\hat{y}$

28. Cross product, component form :

$$\begin{aligned}
\bar{A}\times\bar{B} = & \quad (A_1\hat{x}+A_2\hat{y}+A_3\hat{z})\times(B_1\hat{x}+B_2\hat{y}+B_3\hat{z}) \\
= & \quad A_1B_1\hat{x}\times\hat{x}+A_1B_2\hat{x}\times\hat{y}+A_1B_3\hat{x}\times\hat{z}+A_2B_1\hat{y}\times\hat{x}+A_2B_2\hat{y}\times\hat{y} \\
& \quad +A_2B_3\hat{y}\times\hat{z}+A_3B_1\hat{z}\times\hat{x}+A_3B_2\hat{z}\times\hat{y}+A_3B_3\hat{z}\times\hat{z} \\
= & \quad A_1B_2\hat{x}\times\hat{y}+A_1B_3\hat{x}\times\hat{z}+A_2B_1\hat{y}\times\hat{x}+A_2B_3\hat{y}\times\hat{z}+A_3B_1\hat{z}\times\hat{x}+A_3B_2\hat{z}\times\hat{y} \\
= & \quad A_1B_2\hat{z}+A_1B_3(-\hat{y})+A_2B_1(-\hat{z})+A_2B_3\hat{x}+A_3B_1\hat{y}+A_3B_2(-\hat{x}) \\
= & \quad (A_2B_3-A_3B_2)\,\hat{x}+(A_3B_1-A_1B_3)\,\hat{y}+(A_1B_2-A_2B_1)\,\hat{z} \\
= & \quad \begin{vmatrix} A_2 & A_3 \\ B_2 & B_3 \end{vmatrix}\hat{x}-\begin{vmatrix} A_1 & A_3 \\ B_1 & B_3 \end{vmatrix}\hat{y}+\begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix}\hat{x} \\
= & \quad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}
\end{aligned}$$

29. Coordinates : Rectangular (x,y,z) , cylindrical (r,ϕ,z) , spherical (R,ϕ,θ)

30. Cylindrical coordinate : $r=\sqrt{x^2+y^2}$, $\phi=\tan^{-1}\dfrac{y}{x}$, $x=r\cos\phi$, $y=r\sin\phi$

31. Spherical coordinate : $R=\sqrt{x^2+y^2+z^2}$, $\phi=\tan^{-1}\dfrac{y}{x}$, $\theta=\tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z}$, $x=R\sin\theta\cos\phi$, $y=R\sin\theta\sin\phi$, $z=R\cos\theta$

32. Vector in component form $\bar{V}=V_x\hat{x}+V_y\hat{y}+V_z\hat{z}=V_r\hat{r}+V_\phi\hat{\phi}+V_z\hat{z}=V_R\hat{R}+V_\phi\hat{\phi}+V_\theta\hat{\theta}$

33. Vector magnitude $|\bar{V}|=\sqrt{V_x^2+V_y^2+V_z^2}=\sqrt{V_r^2+V_\phi^2+V_z^2}=\sqrt{V_R^2+V_\phi^2+V_\theta^2}$

34. Euclidean distance:

$$d=\begin{cases} \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2} \\ \\ \sqrt{r_1^2+r_2^2-2r_1r_2\cos(\phi_1-\phi_2)+(z_1-z_2)^2} & =\sqrt{V_R^2+V_\phi^2+V_\theta^2} \\ \\ \sqrt{R_1^2+R_2^2-2R_1R_2\cos\theta_1\cos\theta_2-2R_1R_2\sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)} \end{cases}$$

35. Triple Dot Product :

$$\bar{A}\cdot\left(\bar{B}\times\bar{C}\right)=\bar{A}\cdot\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$=(A_1,A_2,A_3)\cdot\left(\begin{vmatrix} B_2 & B_3 \\ C_2 & C_3 \end{vmatrix},\begin{vmatrix} B_1 & B_3 \\ C_1 & C_3 \end{vmatrix},\begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix}\right)=\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

36. $\bar{A}\cdot\left(\bar{A}\times\bar{B}\right)=\bar{A}\cdot\left(\bar{B}\times\bar{A}\right)=0$

37. Triple Dot Product :

$$\bar{A}\cdot\left(\bar{B}\times\bar{C}\right)=\left(\bar{B}\times\bar{C}\right)\cdot\bar{A}=\bar{B}\cdot\left(\bar{C}\times\bar{A}\right)=\left(\bar{C}\times\bar{A}\right)\cdot\bar{B}=\bar{C}\cdot\left(\bar{A}\times\bar{B}\right)=\left(\bar{A}\times\bar{B}\right)\cdot\bar{C}$$