

Eigenvalues

2013

Given matrix A has eigenvalue λ .

Question: what about eigenvalue of A^2 ?

Answer:

$$Ax = \lambda x$$

$$\iff AAx = A\lambda x$$

$$\iff A^2x = \lambda Ax$$

So

$$A^2x = \lambda^2x$$

So eigenvalue of A^2 is λ^2

Question: what about eigenvalue of A^{-1} ?

Answer:

$$Ax = \lambda x$$

So

$$x = \frac{Ax}{\lambda}$$

Multiply A^{-1} on both side (assume A^{-1} exists)

$$A^{-1}x = A^{-1}\frac{Ax}{\lambda}$$

$$A^{-1}x = \frac{1}{\lambda}x$$

So the eigenvalue of A^{-1} is λ^{-1}

Question: what about eigenvalue of $(A^{-1} - I)$?

Let the eigenvalue of $A^{-1} - I$ be λ'

$$(A^{-1} - I)x = \lambda'x$$

Then

$$A^{-1}x = (\lambda' + 1)x$$

Since the eigenvalue of A^{-1} is λ^{-1} , thus

So

$$\lambda^{-1} = \lambda' + 1$$

$$\lambda' = \lambda^{-1} - 1$$