NAIVE SET THEORY

1. Set

(1) Set $S = \{x : x \in S\}$, a set S is a collection of element x . Those x are belongs to the set S
(2) Equal Set : $A = B$ iff they consist of the same elements
(3) Proper subset : $A \subset B$ and $A \neq B$, A is proper subset of B
(4) Improper subset : $A \subseteq B$ means $A = B$ is not excluded. For all set S , $S \subseteq S$ (S is improper subset of itself)
(5) Complement : for $A \subset B$ (A is a proper subset of B), then for the set $\{x : x \in B, x \notin A\}$ is the complement of A in B , denoted as A^C.
(6) Null set : the complement of improper subset is the null set. $\varnothing = \{\}$. $\varnothing$ is a subset for all set S : $\varnothing$ is a proper subset for all set $S \neq \varnothing$, and $\varnothing$ is a improper subset for all set $S = \varnothing$.
(7) Universal set : $U \neq \varnothing$ is a universal set that subset are under consideration. Some universal set can not explicitly defined, such as $\mathbb{R}$.
(8) Intersection : $A \cap B = \{x : x \in A \text{ and } x \in B\}$
(9) Union : $A \cup B = \{x : x \in A \, \text{or} \, x \in B\} = \{x : x \in A \, \text{alone or} \, x \in B \text{ alone or } x \in A \cap B\}$
(10) Disjoint : A and B are disjoint iff they have no common element , $A \cap B = \varnothing$
(11) Difference : $A - B = \{x : x \in A, \, x \notin B\}$
(12) $A - B = A \cap B^C = B^C - A^C$: $A - B = \{x : x \in A, \, x \notin B\} = \{x : x \in A \text{ and } x \in B^C\} = \{x : x \notin A^C \, , \, x \in B^C\}$
(13) $A - B = \varnothing$ iff $A \subseteq B$: if $A - B = \varnothing$ then $A \cap B^C = \varnothing$(by12) so A and B^C are disjoint, and since B and B^C is disjoint with $B \cup B^C = U$, thus $A \subseteq B$. if $A \subseteq B$, then $A \cap B^C = \varnothing$ and thus $A \cap B^C = A - B = \varnothing$
(14) $A - B = A$ iff $A \cap B = \varnothing$: if $A - B = A$, then $A \cap B^C = A$, thus $A \subseteq B^C$ and thus $A \cap B = \varnothing$ (A and B are disjoint). If $A \cap B = \varnothing$, then A and B are disjoint, so $A - B^C = \varnothing$ (by12) and thus $A \subseteq B^C$ (by13), thus $A \cap B^C = A$ and $A - B = A$ (by12)