Standardized Random Variable $\mathcal{Z}$

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For a random variable $X(\mu,\sigma)$, the standardization is

$$Z = \frac{X-\mu}{\sigma}$$

Why Standardize

- Before standardization , distribution of X depends on μ and σ, after standardization , distribution of Z does not depend on µ and σ anymore. This is good because usually these quantities are generally unknown.

The Mean and variance of Z

Recall,

$$\mu_X = E\left[X\right] = \sum x_i f(x_i) \qquad \sum f(x_i) = 1 \qquad E\left[aX+bY\right] = aE\left[X\right] + bE\left[Y\right]$$

Thus, the mean of Z is

$$\mu_Z = E\left[Z\right] = E\left[\frac{X-\mu}{\sigma}\right] = \frac{1}{\sigma}E\left[X-\mu\right] = \frac{1}{\sigma}E\left[X\right] - \frac{1}{\sigma}E\left[\mu\right]$$

$$= \frac{1}{\sigma}\mu - \frac{1}{\sigma}\sum f(x_i)\mu = \frac{1}{\sigma}\mu - \frac{1}{\sigma}\mu_i \underbrace{\sum f(x_i)}_{1} = \frac{1}{\sigma}\mu - \frac{1}{\sigma}\mu_i = 0$$

And recall that

$$Var\left(X\right) = \sigma^2 = E\left[\left(x_i-\mu\right)^2\right] = \sum \left(x_i-\mu\right)^2 f(x_i) \qquad Var\left(aX+k\right) = a^2 Var(X)$$

Thus, the variance of Z is

$$Var(Z) = Var\left(\frac{X-\mu}{\sigma}\right) = Var\left(\frac{X}{\sigma} - \frac{\mu}{\sigma}\right) = \frac{1}{\sigma^2}Var\left(X\right) = \frac{1}{\sigma^2}\sigma^2 = 1$$

Thus , the standardized random variable Z , it has mean of zero and variance of 1 , i.e.

$$Z\left(0,1\right)$$

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