Lists of Transforms

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1 Definitions

1.1 Euler's Formula

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \sin \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2j}$$

1.2 Phasor

$$A\cos(\omega t + \phi) = Re\left\{Ae^{j(\omega t+\phi)}\right\} = Re\left\{Ae^{j\phi}e^{j\omega t}\right\}$$

1.3 Laplace

$$F(s) = \mathcal{L}\left\{f(t)\right\} = \int_0^\infty f(t)e^{-st}dt$$

- Time domain changed into s-domain
- For Causal System : $f(t) = 0$ for $t < 0$.
- For non-causal input $f(t)$, the Laplace Transform is formed by multiply the input with Heaviside Unit Step to causalize it.
- $F(s) = \mathcal{L}\left\{f(t)\right\} = \int_0^\infty f(t)u(t)e^{-st}dt$
- Application of $\mathcal{L}$: Turning complicated *linear time invariant ordinary differential equation* into simple algebra problem

1.4 Fourier (Continuous)

$$F(\omega) = \mathcal{F}\left\{f(x)\right\} = \int_{-\infty}^\infty f(x)e^{-j\omega x}dx$$

- Time or Spatial domain (periodic or non-periodic) into frequency ω domain
- For any system (periodic or non-periodic , causal or non-causal)
- Application of $\mathcal{F}$: Breaking complicated spatial/time domain function/signal into simple basic building block

1.5 Zeta

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Sequence or series in digital domain into z domain
- For digital system / discrete system / series
- Application of $\mathcal{Z}$: Turning complicated recurrence equation / difference equation into simple algebra in terms of z^{-1}

1.6 Hilbert

$$\widehat{x}(t) = \mathcal{H}\{x(t)\} = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

- $\widehat{x}(t) \in \mathbb{R}$
- Hilber Transform can be treated as convolution of $x(t)$ with $\dfrac{1}{\pi t}$, which is $h(t)$
- A real function / signal $x(t)$ and its Hilbert Transfrom can form a *analytic signal* : $x_+(t) = x(t) + j\widehat{x}(t) \in \mathbb{C}$

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