The Analog Modulations
General Overview

December 18, 2012

1 Amplitude Modulation - Simple AM

Basic Form of signal

$$v_{AM}(t) = [A_C + m(t)] \cos \omega_c t$$

For monotone message $m(t) = A_m \cos \omega_m t$

$$v_{AM}(t) = [A_C + A_m \cos \omega_m t] \cos \omega_c t = A_c [1 + m \cos \omega_m t] \cos \omega_c t$$

The modulation index

$$m = \frac{A_m}{A_c}$$

Generation method : Frequency mixer and adder

De-modulation method : By frequency mixer + filter

$$v_{De-AM}(t) = v_{AM} \cdot [A_C \cos \omega_c t] = [A_C + m(t)] A_c \cos^2 \omega_c t = \left[A_C^2 + A_c m(t)\right] \frac{1 + \cos 2\omega_c t}{2}$$

$$= \underbrace{\frac{A_c^2}{2}}_{\text{Filter by Capacitor}} + \frac{A_c}{2} m(t) + \underbrace{\frac{[A_C^2 + A_c m(t)] \cos 2\omega_c t}{2}}_{\text{Filtered by LP}} \rightarrow \frac{A_c}{2} m(t)$$

About the bandwidth

For message with bandwidth ω_m

$$m(t) \longleftrightarrow M(\omega_m)$$

Then by Fourier Transform effect of $\cos \omega_c t$

$$BW_{AM} = 2\omega_m$$

1.1 Improvement on A_c

Improvement can be made by select the A_c as 2 , then

$$v_{De-AM} = m(t) = \text{Original signal}$$

1.2 Power Improvement : DSBSC

Simple AM is also called Double Sideband larger Carrier AM, the power efficiency is low, so use another modulation

Double Sideband suppressed carrier AM

$$v_{AM}(t) = [A_C + m(t)] \cos \omega_c t \quad \xrightarrow{\text{Drop the Carrier}} \quad v_{DSBSC}(t) = m(t) \cos \omega_c t$$

The generation method is simply just a frequency mixer

De-modulation method : By frequency mixer + filter

$$v_{De-DSBSC}(t) = v_{DSBSC} \cdot [\cos \omega_c t] = m(t) A_c \cos^2 \omega_c t = m(t) \frac{1 + \cos 2\omega_c t}{2}$$

$$= \frac{1}{2} m(t) + \underbrace{\frac{m(t) \cos 2\omega_c t}{2}}_{\text{Filtered by LP}} \rightarrow \frac{1}{2} m(t) = \frac{1}{2} \text{original signal}$$

Remark
Since A_c is droped out

$$m = \frac{A_m}{A_c} = \infty$$

Thus modulation index is meaningless in suppressed carrier AM

But for the bandwidth, it is still the same

$$BW_{DSBSC} = 2\omega_m$$

1.3 Further improvement in power and bandwidth : SSBSC and VSBSC

Further improvement can be made by using : Single Sideband Suppressed Carrier AM
But ideal filter is very expensive, so when using a not-so-ideal filter, the VSBSC signal is generated

$$v_{DSBSC} \xrightarrow{\substack{\text{Ideal} \\ \text{BPF}}} v_{SSBSC} \qquad v_{DSBSC} \xrightarrow{\substack{\text{Non-Ideal} \\ \text{BPF}}} v_{VSBSC}$$

Thus, the bandwidth is

$$BW_{SSB} = \omega_m$$

2 Angle Mpodulations : FM and PM

General Form

$$v(t) = A_c \cos(\omega_c t + \phi(t))$$

Where

$$\phi_f(t) = k_f \int m(t)dt \qquad \phi_p(t) = k_p m(t)$$

Thus

$$v_{FM}(t) = A_c \cos\left(\omega_c t + k_f \int m(t)\right) \qquad v_{PM}(t) = A_c \cos(\omega_c t + k_p m(t))$$

For monotone message, $m(t) = A_m \cos \omega_m t$

$$v_{FM}(t) = A_c \cos\left(\omega_c t + \frac{k_f A_m}{\omega_m} \sin(\omega_m t)\right) \qquad v_{PM}(t) = A_c \cos(\omega_c t + k_p A_m \cos \omega_m t)$$

Where the modulation index are

$$\beta_f = \frac{k_f A_m}{\omega_m} \qquad \beta_m = k_p A_m$$

Thus the waveform can be represented as

$$v_{FM}(t) = A_c \cos(\omega_c t + \beta_f \sin \omega_m t) \qquad v_{PM}(t) = A_c \cos(\omega_c t + \beta_p \cos \omega_m t)$$

2.1 Bessel Expansion and Bandwidth

The term

$$\cos(\theta + \beta \sin \phi)$$

can be expanded using Bessel Function.

$$v_{FM}(t) = \sum_{n=-\infty}^{\infty} J(\beta_n) \cos(\omega_c t + n\omega_m t) \qquad v_{PM}(t) = \sum_{n=-\infty}^{\infty} J(\beta_n) \cos(\omega_c t + n\omega_m t)$$

The amplitude of the harmonics can be found by using Bessel Table.

When the n^{th} harmonic power is less than 1% of the unmodulated power, that term can be ignored.

Since the power of a sinusoidal $\propto$amplitude^2, so

$$\text{The } n^{th} \text{ harmonic can be ignored if } |J(\beta_n)| < 0.01$$

Then, in that case, the bandwidth of the wave is

$$BW_{FM/PM} = 2n\omega_m$$

The bandwidth of FM/PM signal can also be found using Carson's rule

$$BW_{FM/PM} = 2(\beta + 1)\omega_m = 2(\omega_c + \omega_m)$$

2.2 Narrow Band wave and the generation

When $\beta < 0.2$, it can be treated as Narrow Band signal
Then the signal

$$v_{FM}(t) = A_c \cos(\omega_c t + \beta_f \sin \omega_m t) \qquad v_{PM}(t) = A_c \cos(\omega_c t + \beta_p \cos \omega_m t)$$

Expand by $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$v_{FM}(t) = A_c \cos(\omega_c t) \cos(\beta_f \sin \omega_m t) - \sin \omega_c t \sin(\beta_f \sin \omega_m t)$$

$$v_{PM}(t) = A_c \cos(\omega_c t) \cos(\beta_p \cos \omega_m t) - \sin(\omega_c t) \sin(\beta_p \cos \omega_m t)$$

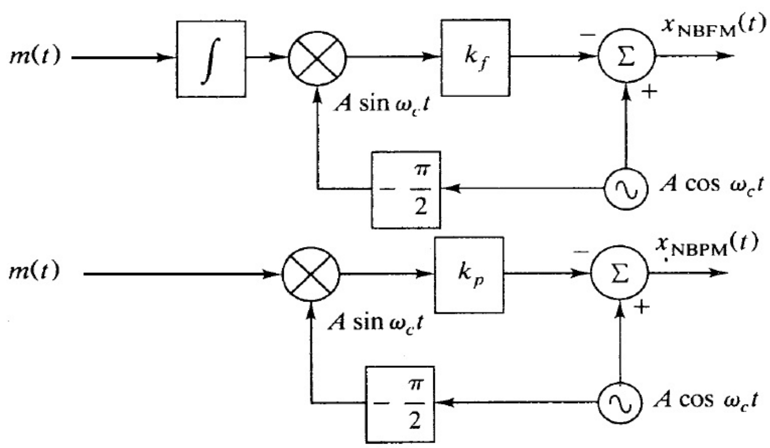
Since $\beta < 0.2$, it is small, so

$$\cos \beta \left(\underbrace{\sin \omega_m t}_{<1} \right) \approx \cos 0 = 1 \qquad \sin \theta \approx \theta \qquad \text{when small}\theta$$

$$v_{FM}(t) = A_c \cos(\omega_c t) - \sin \omega_c t \cdot \beta_f \sin \omega_m t = A_c \cos(\omega_c t) - \sin \omega_c t \cdot k_f \int m(t)dt$$

$$v_{PM}(t) = A_c \cos(\omega_c t) - \sin(\omega_c t) \beta_p \cos \omega_m t = A_c \cos(\omega_c t) - \sin(\omega_c t) k_p m(t)$$

Thus the generation of narrow band FM & PM can be achieve by using Hilbert Transfrom, integrator, amplifier and adder.

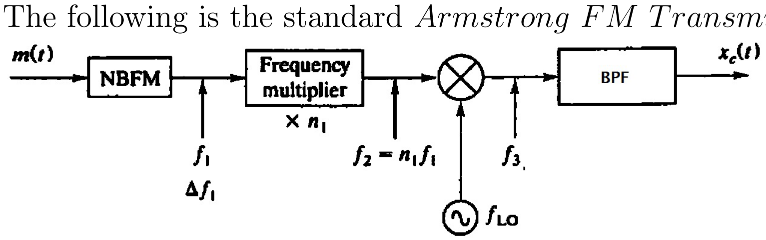


2.3 Wide band FM PM and the generation

When $\beta > 1$, it can be considered to be WBFM, WBPM

The generation of WB wave can use frequency multiplier, or VCO (Voltage Controlled Oscillator)

The following is the standard *Armstrong FM Transmitter*, which use the frequency multiplier



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