Amplitude Modulation

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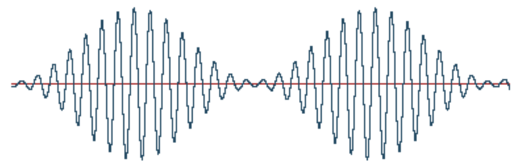
Reference

Hwei P. Hsu *Analog and Digital Communication*



Summary

	Equation
Message	General $m(t)$ Monotone $A_m \cos \omega_m t$
Carrier	Large Carrier $A_c \cos \omega_c t$ Unity $\cos \omega_c t$
Simple AM DSB-LC	General $(A_c + m(t)) \cos \omega_c t = A_c \cos \omega_c t + m(t) \cos \omega_c t$ Monotone $A_c \cos \omega_c t + \frac{mA_c}{2} \cos (\omega_c - \omega_m) t + \frac{mA_c}{2} \cos (\omega_c + \omega_m) t$
DSB-SC	General $m(t) \cos \omega_c t$ Monotone $\frac{mA_c}{2} \cos (\omega_c - \omega_m) t + \frac{mA_c}{2} \cos (\omega_c + \omega_m) t$
SSB / VSB	Monotone$\frac{mA_c}{2} \cos (\omega_c \pm \omega_m) t$-

	Bandwidth	Advantage	Disadvantage
Simple AM DSB-LC	$2\omega_m$	\$	Low η
DSB-SC	$2\omega_m$	Highη	\$\$\$
SSB/VSB	ω_m	High η Small BW	\$\$\$\$\$

1 Introduction

Message signal and Carrier Signal in time domain

$$\begin{cases} \text{Monotone} & A_m \cos\omega_m t = \frac{A_m}{2}\left(e^{j\omega_m t} + e^{-j\omega_m t}\right) \\ \text{General} & m(t) \end{cases} \qquad \begin{cases} \text{Unity Carrier} & \cos\omega_c t = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) \\ \text{Larger Carrier} & A_c \cos\omega_c t = \frac{A_c}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) \end{cases}$$

- 2 frequency component in same amplitude
- Negative frequency is not real, just mathematical
- $\omega_c > \omega_m$, if not ,Aliasing

1.1 Continuous Wave Modulation

CW modulation is just a multiplication, for montone message and larger carrier :

$$(A_m \cos\omega_m t) \times (A_c \cos\omega_c t)$$

By identity $\cos A \cos B = \dfrac{\cos(A+B) + \cos(A-B)}{2}$

$$\frac{A_m A_c}{2}\left\{\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t\right\} = \frac{A_m A_c}{4}\left\{e^{j(\omega_c-\omega_m)t} + e^{-j(\omega_c-\omega_m)t} + e^{j(\omega_c+\omega_m)t} + e^{-j(\omega_c-\omega_m)t}\right\}$$

- One cosine term has 2 frequency spectral line, so 2 cosine terms have 4 spectral line
- Frequency spectra line translated
- Negative frequency is not real, it is only mathematical, so although it has 4 spectra, it actually has only 2 spectra

1.2 The signal's Fourier Transfroms

For general message

$$m(t) \overset{\mathcal{F}}{\longleftrightarrow} M(j\omega_m)$$

CW Modulated signal of general message with large carrier

$$m(t)A_c \cos\omega_c t = \frac{A_c}{2} m(t)\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) \overset{\mathcal{F}}{\longleftrightarrow} \frac{A_c}{2}\left[M(j\omega_m + j\omega_c) + M(j\omega_m - j\omega_c)\right]$$

2 Amplitude Modulation

There are 3 to 4 basic types of AM modulation : Simple AM (DSB-LC) , DSB-SC, SSB, and VSB

2.1 Simple AM signal in Time Domain

AM : Message signal "embed" into the amplitude of carrier

Simple AM is also called : Double Sideband Large Carrier (DSB-LC) Modultion

For general message signal $m(t)$:

$$x_{AM}(t) = (A_c + m(t))\cos\omega_c t$$

For simple monotone message signal $m(t) = A_m \cos\omega_m t$:

$$x_{AM}(t) = (A_c + A_m \cos\omega_m t)\cos\omega_c t = A_c \cos\omega_c t + A_m \cos\omega_m t \cos\omega_c t$$

$$= A_c \cos\omega_c t + A_c m \cos\omega_m t \cos\omega_c t \qquad \text{where } m = \frac{A_m}{A_c}$$

By $\cos A \cos B = \dfrac{\cos(A+B) + \cos(A-B)}{2}$

$$= A_c \cos\omega_c t + \frac{mA_c}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$= A_c \cos\omega_c t + \frac{mA_c}{2}\cos(\omega_c + \omega_m)t + \frac{mA_c}{2}\cos(\omega_c - \omega_m)t$$

Simple AM / DSB-LC modulated signal is thus

$$\begin{cases} x_{AM}(t) = (A_c + m(t))\cos\omega_c t = A_c \cos\omega_c t + m(t)\cos\omega_c t & \text{General} \\ x_{AM}(t) = (A_c + m(t))\cos\omega_c t = A_c \cos\omega_c t + \dfrac{mA_c}{2}\cos(\omega_c + \omega_m)t + \dfrac{mA_c}{2}\cos(\omega_c - \omega_m)t & \text{Monotone Message} \end{cases}$$

- Envelope of modulated signal will follow the message signal
- $m = \dfrac{A_m}{A_c} 100\% \quad m = 100\% \iff A_m = A_c$
- $m > 100\% \iff$ overmodulation $A_m > A_c$
- Requirements for AM / DSB-LC : $A_c > A_m$, $\omega_c \gg \omega_m$

2.2 Simple AM signal in Frequency Domain

For general message signal

$$m(t) \overset{\mathcal{F}}{\longleftrightarrow} M(j\omega_m) \qquad\qquad Bandwidth = [0\,,\,\omega_m] = \omega_m$$

The simple AM signal

$$x_{AM}(t) = (A_c + m(t))\cos\omega_c t = A_c\cos\omega_c t + m(t)\cos\omega_c t$$

The AM signal's Fourier Transform

$$\mathcal{F}\{x_{AM}(t)\} = \mathcal{F}\{A_c\cos\omega_c t\} + \mathcal{F}\{m(t)\cos\omega_c t\}$$

$$= A_c\mathcal{F}\{\cos\omega_c t\} + \frac{1}{2}\mathcal{F}\left\{m(t)\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)\right\} = A_c\mathcal{F}\{\cos\omega_c t\} + \frac{1}{2}\mathcal{F}\left\{m(t)e^{j\omega_c t}\right\} + \frac{1}{2}\mathcal{F}\left\{m(t)e^{-j\omega_c t}\right\}$$

$$= A_c\int_{-\infty}^{\infty}\cos\omega_c t e^{-j\omega t}dt + \frac{1}{2}\int_{-\infty}^{\infty} m(t)e^{-j(\omega-\omega_c)t}dt + \frac{1}{2}\int_{-\infty}^{\infty} m(t)e^{-j(\omega+\omega_c)t}dt$$

As $m(t) \overset{\mathcal{F}}{\longleftrightarrow} M(j\omega_m)$ thus $m(t)e^{\pm jat} \overset{\mathcal{F}}{\longleftrightarrow} M(j\omega \pm ja)$

$$= \frac{A_c}{2}\int_{-\infty}^{\infty}\left(e^{-j(\omega-\omega_c)t} + e^{-j(\omega+\omega_c)t}\right)dt + \frac{1}{2}M(j\omega - j\omega_c) + \frac{1}{2}M(j\omega - j\omega_c)$$

$$X_{AM}(j\omega) = \text{-}\frac{A_c}{2}\delta(\omega \pm \omega_c) + \frac{1}{2}M(j\omega - j\omega_c) + \frac{1}{2}M(j\omega - j\omega_c)$$

- Bandwidth is twice of original bandwidth : $BW_{AM} = 2W = 2\omega_m$
- AM wave contains 2 sideband with bandwidth of each band as $W_{LSB} = W_{USB} = W = \omega_m$

2.3 Double-Sideband Suppressed-Carrier Modulation DSBSC

Recall, simple AM / DSB-LC for $m(t) = A_m \cos \omega_m t$

$$\begin{cases} x_{AM}(t) = (A_c + m(t)) \cos \omega_c t = A_c \cos \omega_c t + m(t) \cos \omega_c t & \text{General} \\ x_{AM}(t) = A_c \cos \omega_c t + \dfrac{mA_c}{2} \cos(\omega_c + \omega_m)\, t + \dfrac{mA_c}{2} \cos(\omega_c - \omega_m)\, t & \text{Monotone Message} \end{cases}$$

- Carrier contains no info, so suppresse it to enhance power efficiency

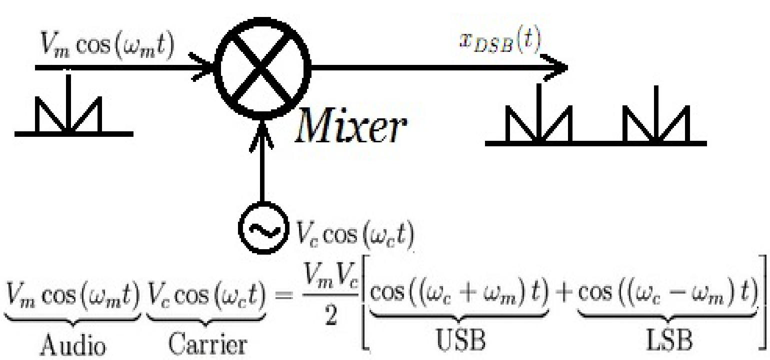
After dropping the carrier term,

$$\begin{cases} x_{DSB-SC}(t) = m(t) \cos \omega_c t = m(t) \cos \omega_c t & \text{General} \\ x_{DSB-SC}(t) = \dfrac{mA_c}{2} \cos(\omega_c + \omega_m)\, t + \dfrac{mA_c}{2} \cos(\omega_c - \omega_m)\, t & \text{Monotone Message} \end{cases}$$

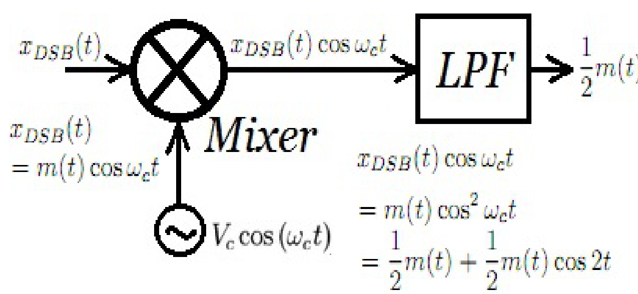
For general message signal $m(t) \overset{\mathcal{F}}{\longleftrightarrow} M(t)$, the Fourier Transform is

$$\mathcal{F}\left\{x(t)_{\text{DSB-SC}}\right\} = \mathcal{F}\left\{m(t) \cos \omega_c t\right\} = \mathcal{F}\left\{m(t) \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right\} = \frac{1}{2}\left[M(j\omega - j\omega_c) + M(j\omega + j\omega_c)\right]$$

2.4 Generation & Demodulation of DSB signal

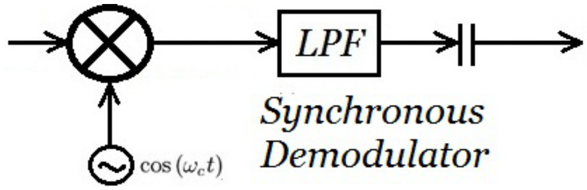


DSBSC Modulation



DSBSC Demodulation

For Simple AM / DSB-LC



- After mixing : -$x_{AM}(t)\cos\omega_c t = [A_c + m(t)]\cos^2\omega_c t$
$$= [A_c + m(t)]\frac{1+\cos 2\omega_c t}{2} = \frac{A_c + m(t)}{2} + \frac{A_c + m(t)}{2}\cos\omega_c t$$
- After passing LPF : $\frac{A_c}{2} + \frac{m(t)}{2}$
- After passing capacitor to block the DC : $\frac{m(t)}{2}$
- Thus, with suitable amplifier, the original signal can be recovered.
- * Then this demodulator works $\forall A_c$

Since carrier is suppressed, so $A_m \to 0$, and thus $m = \frac{A_m}{A_c} \to \infty$ so modulation index is meaningless for DSB-SC signal.

2.5 Single-Sideband Suppressed Carrier

To improve power efficiency, further dropping one sideband

For monotone signal, the DSB-SC signal is

$$x_{DSB-SC}(t) = \frac{mA_c}{2}\cos(\omega_c + \omega_m)t + \frac{mA_c}{2}\cos(\omega_c - \omega_m)t$$

After dropping one sideband

$$\begin{cases} \begin{matrix}\text{After LPF}\\ \text{LSB}\end{matrix} & x_{SSB-LSB}(t) = \frac{mA_c}{2}\cos(\omega_c - \omega_m)t \\ \\ \begin{matrix}\text{After HPF}\\ \text{HSB}\end{matrix} & x_{SSB-USB}(t) = \frac{mA_c}{2}\cos(\omega_c + \omega_m)t \end{cases}$$

Vestigial Sideband Signal

Since SSB require a **very sharp cut-off filter** to remove one sideband, such filter is not easy to implement

Thus, the requirement is relaxed by allowing a vestige part : Vestigial Sideband signal

3 Power of the AM Signal

3.1 Review of Root-Mean-Square value

For a function $A\cos\omega t$, the RMS value is :

$$RMS\left(A\cos\omega t\right) = \sqrt{\frac{1}{T}\int_0^T \left(A\cos\omega t\right)^2 dt} = A\sqrt{\frac{1}{T}\int_0^T \cos^2\omega t dt} = A\sqrt{\frac{1}{T}\int_0^T \frac{1+\cos 2\omega t}{2}dt}$$

Since $\cos\theta$ is orthogonal to 1, so the second integral is zero

$$= A\sqrt{\frac{1}{T}\int_0^T \frac{1}{2}dt + \frac{1}{2T}\underbrace{\int_0^T \cos 2\omega t dt}_{0}} = A\sqrt{\frac{T}{2T}} = \frac{A}{\sqrt{2}}$$

So the RMS value of a function of $A\cos\omega t$ is $\dfrac{A}{\sqrt{2}}$

Then , recall that simple AM / DSB-LC signal of montone message has the form

$$x_{AM}(t) = \left(A_c + m(t)\right)\cos\omega_c t = A_c\cos\omega_c t + \frac{A_c m}{2}\cos\left(\omega_c - \omega_m\right)t + \frac{A_c m}{2}\cos\left(\omega_c + \omega_m\right)t$$

Then the RMS value is

$$x_{AM,RMS} = \frac{A_c}{\sqrt{2}} + \frac{A_c m}{2\sqrt{2}} + \frac{A_c m}{2\sqrt{2}}$$

Thus the total power , by $P = \dfrac{A^2}{R}$, is

$$P_{T,AM} = \frac{A_c^2}{2R} + \frac{A_c^2 m^2}{8R} + \frac{A_c^2 m^2}{8R}$$

The power used to transmitte information for simple AM is thus :

$$\eta_{AM} = \frac{P_{Info}}{P_T} = \frac{\frac{A_c^2 m^2}{8R} + \frac{A_c^2 m^2}{8R}}{\frac{A_c^2}{2R} + \frac{A_c^2 m^2}{8R} + \frac{A_c^2 m^2}{8R}} = \frac{\frac{m^2}{4} + \frac{m^2}{4}}{1 + \frac{m^2}{4} + \frac{m^2}{4}} = \frac{2m^2}{4+2m^2} = \frac{m^2}{2+m^2}$$

When $m = 1$

$$\eta_{AM} = \frac{1}{3} \quad \Longleftrightarrow \quad \frac{2}{3} = 66.6\%\text{Power Lost}$$

Therefore, simple AM signal is not power-efficienct.

$$\eta_{AM} < 33.3\% = \sup\eta_{AM}$$

In summary, for simple AM / DSB-LC signal,

- The efficiency is limited to 33%
- The carrier signal is present even if nothing is being transmitted
- The circuitary is relatively simple (only envelop detector is required !)
- Bandwidth is $2\omega_m$

For Dobule Sideband Suppressed Carrier of montone message , the wave form is

$$x_{DSB}(t) = x_{AM}\,\text{(Without Carrier)} = \frac{A_c m}{2}\cos\left(\omega_c - \omega_m\right)t + \frac{A_c m}{2}\cos\left(\omega_c + \omega_m\right)t$$

Thus, the RMS value is

$$x_{DSB,RMS} = \frac{A_c m}{2\sqrt{2}} + \frac{A_c m}{2\sqrt{2}}$$

Thus, the Total power is

$$P_{T,DSB} = \frac{A_c^2 m^2}{8R} + \frac{A_c^2 m^2}{8R}$$

And hence, the power efficiency is

$$\eta_{DSB} = \frac{P_{Info}}{P_T} = \frac{P_T}{P_T} = 100\%\ \text{(Ideal)}$$

The power efficiency of DSB singal is very good, but the tradeoff is it require relatively expensive circuitry in the receiver

In summary, for DSB signal

- It have must higher power efficiency ($\sim 100\%$)
- But it has same bandwidth as simple AM , $2\omega_m$
- It require relatively expensive circuitry in the receiver

For SSB singal, a sideband filter, either high pass or low pass, is concatenated to the receiver circuit. For Single Sideband Suppressed Carrier of montone message, the wave form is

$$\text{-}x_{SSB}(t) = \frac{A_c m}{2}\cos\left(\omega_c \pm \omega_m\right)t$$

Thus, the RMS value is

$$x_{DSB,RMS} = \frac{A_c m}{2\sqrt{2}}$$

Thus, the Total power is

$$P_{T,SSB} = \frac{A_c^2 m^2}{8R}$$

And hence, the power efficiency is

$$\eta_{SSB} = \frac{P_{Info}}{P_T} = \frac{P_T}{P_T} = 100\%\ \text{(Ideal)}$$

In summary, for SSB signal,

- It has high power efficiency ($\sim 100\%$)
- It has relatively most expensive circuitry (An extra sideband filter)
- It cut bandwidth in half, $BW_{SSB} = \omega_m$

$-END-$