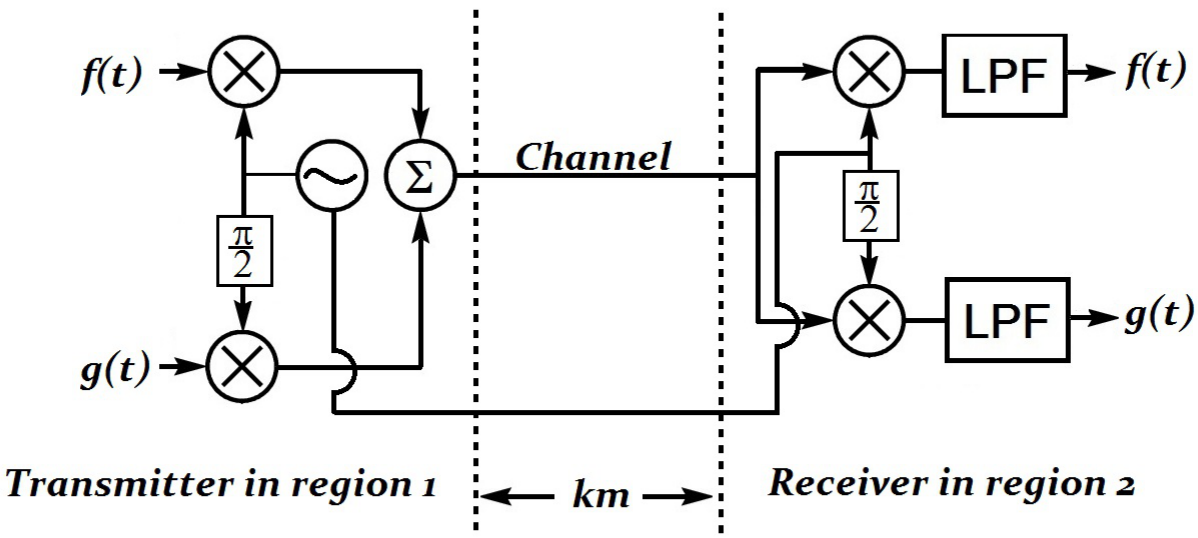
Quadrature Amplitude Modulation : Introduction

Ang Man Shun

December 20, 2012

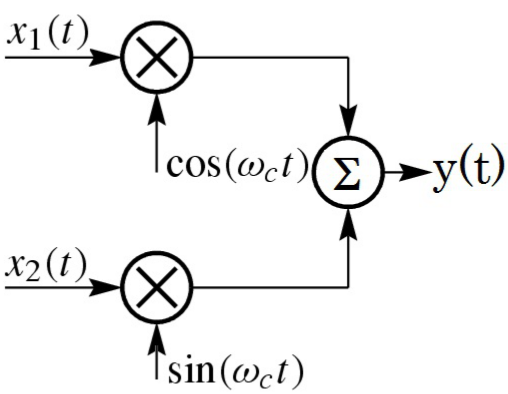


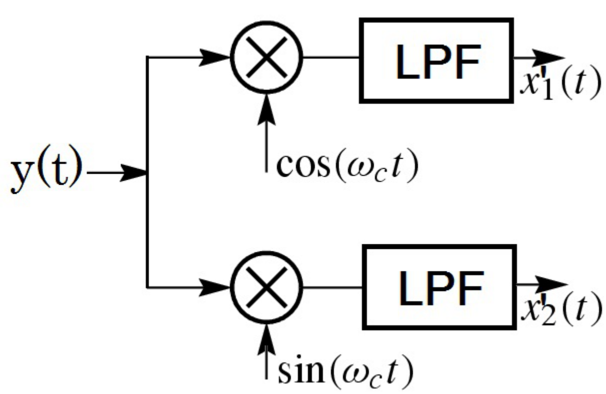
To send multiple signals at one time, one way is to use QAM.

Some trigonometric identities

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin\theta\cos\theta = \frac{\sin 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

1. QAM structure





Consider the $y(t)$ in the transimitter

$$y(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$$

Consider the upper signal in the receiver, the signal is

$$y(t) \cdot \cos \omega_c t = (x_1(t) \cos \omega_c t + x_2(t) \sin \omega_c t) \cos \omega_c t = x_1(t) \cos^2 \omega_c t + x_2(t) \sin \omega_c t \cos \omega_c t$$

Use those identities, the signal is

$$x_1(t) \left(\frac{1 + \cos 2\omega_c t}{2} \right) + x_2(t) \frac{\sin 2\omega_c t}{2} = \frac{1}{2} x_1(t) + \frac{1}{2} \cos 2\omega_c t \, x_1(t) + \frac{1}{2} x_2(t) \sin 2\omega_c t$$

After passing the low pass filter

$$x_1'(t) = \frac{1}{2} x_1(t)$$

Consider the lower signal in the receiver, the signal is

$$y(t) \cdot \sin \omega_c t = (x_1(t) \cos \omega_c t + x_2(t) \sin \omega_c t) \sin \omega_c t = x_1(t) \sin \omega_c t \cos \omega_c t + x_2(t) \sin^2 \omega_c t$$

$$= x_1(t) \frac{\sin 2\omega_c t}{2} + x_2(t) \frac{1 - \cos 2\omega_c t}{2} = \frac{x_2(t)}{2} + x_1(t) \frac{\sin 2\omega_c t}{2} - x_2(t) \frac{\cos 2\omega_c t}{2}$$

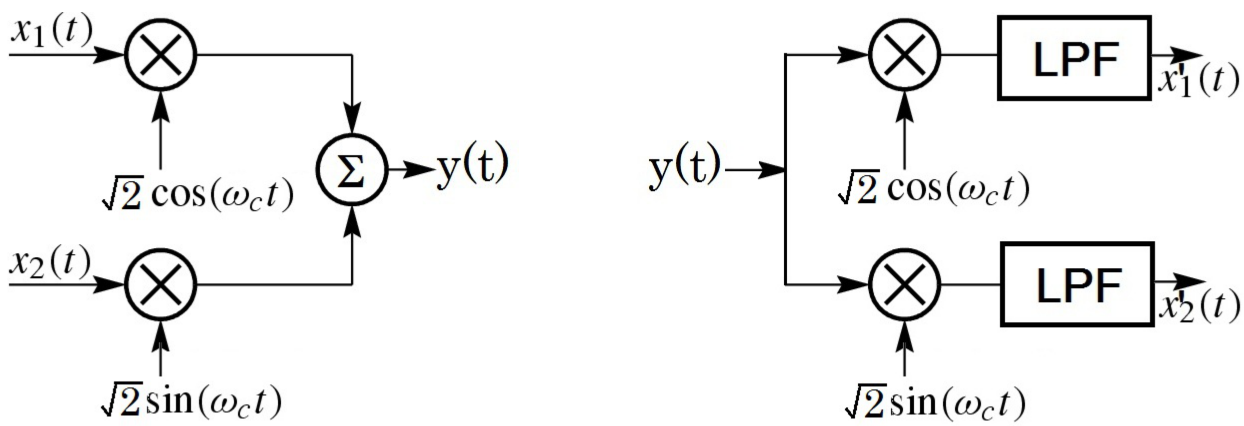
So after passing the LPF

$$x_2'(t) = \frac{x_2(t)}{2}$$

i.e.

$$\begin{cases} x_1'(t) = \dfrac{x_1(t)}{2} \\ x_2'(t) = \dfrac{x_2(t)}{2} \end{cases}$$

Improved QAM



This time the output of the transmitter is

$$y(t) = \sqrt{2} \left[x_1(t) \cos \omega_c t + x_2(t) \sin \omega_c t \right]$$

The signals in receiver are now

$$\begin{cases} \text{Upper} \quad y(t)\sqrt{2} \cos \omega_c t \\ \text{Lower} \quad y(t)\sqrt{2} \sin \omega_c t \end{cases} = \begin{cases} x_1(t) 2 \cos^2 \omega_c t + x_2(t) 2 \sin \omega_c t \cos \omega_c t \\ x_1(t) 2 \cos \omega_c t \sin \omega_c t + x_2(t) 2 \sin^2 \omega_c t \end{cases}$$

$$= \begin{cases} x_1(t) + x_1(t) \cos 2\omega_c t + x_2(t) \sin 2\omega_c t \\ x_2(t) - x_2(t) \cos 2\omega_c t + x_1(t) \sin 2\omega_c t \end{cases}$$

After passing low pass filter

$$x_1'(t) = x_1(t) \qquad x_2'(t) = x_2(t)$$

Thus, this simple QAM is make use of the property of sin-cos function to achieve the objective : sending 2 signal using one channel.

−END−