Signal Acquisition : An overview

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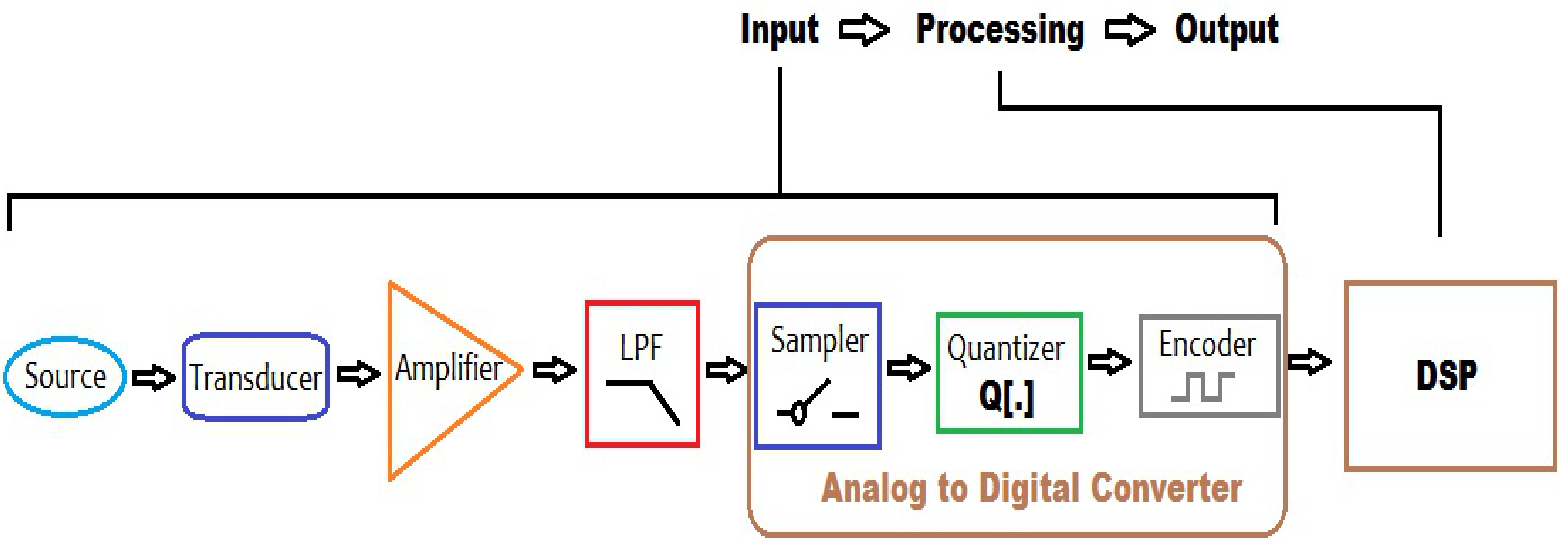
There are 2 type of signal : analog and digital. Digital signals have some advantage : Noise resistance, easy to store, easy to transport.

But the physical world is basically analog. (Let's ignore the Quantum Physics that turns the physical world into pseudo-analog)

Thus, to get the advantage of digital processing, signal acquisition system, is required.

Consider a general analog signal $s(t)$ come from a source S , how to get the digital version of such signal ?

Let's have a look of a generic processing system



A generic system consist of 3 part : Input, Processing, and Output. Here we only talk about input.

1 The general overview

First the source S should has some physical properties that are being well understood, this is the job of the physicist / hardware engineer

Then by using different kind of sensor / transducer, the signal from the source can be "converted" into someform of electrical signal.

Then, the signal obtained from primary transducer need to be amplified. The word "amplification" here does not necessarily means "enlargement", it can also means "deminish"

After amplification, the amplitude of the signal is being adjusted into suitable range, but the bandwidth is not. By $Shannon\,Nyquist\,Sampling\,Theorem$, to prevent aliasing, a LPF "Low Pass Filter" is needed, this filter sometime also called as "Anti-aliasing filter".

After suitable adjustment of amplitude and bandwidth, the signal is now ready to be converted into digital form. To convert a analog signal into digital form, the signal first is fire into a sampler. The output of the sampler is a series of impulses with different height.

The series of impulses have different height, it is still analog in nature. Since analog means real number in mathematics, which has infinite amount of "levels" (such as, you can have value of 3.3325, 0.12433, -0.984531001, π , 0.00000004 ,etc). But digital representations only have finite discrete value , like natural number (only integers are allowed, you can have 1, 4, -5, -99, but not 0.56, 2.44). In this case, a Quantizer is used to *quantize* the continuous analog signal pulses into a finite digital levels. Some inforamtion is loss because the quantization process **introduce quantization error.**

After quantization, the signal now are within suitable amplitude, suitable bandwidth, suitable representable range. But since machine only understand binary value, thus encoding is needed to turn numerical value into binary digits (bits). An encoder perform this function.

After encoding, the signal are now ready go carry out digital signal processing, which is not discussed here.

2 The details

2.1 Sources and Transducer

This part is a little bit physical.

Transducer, (or using another word "sensors") is devices that convert changing physical variables into other form of physical variables.

For example, form high school physics, the Resistance-Temperature Relation of a metal rod can be expressed as

$$R(t) = R_0 (1 + \alpha T)$$

Or more correctly

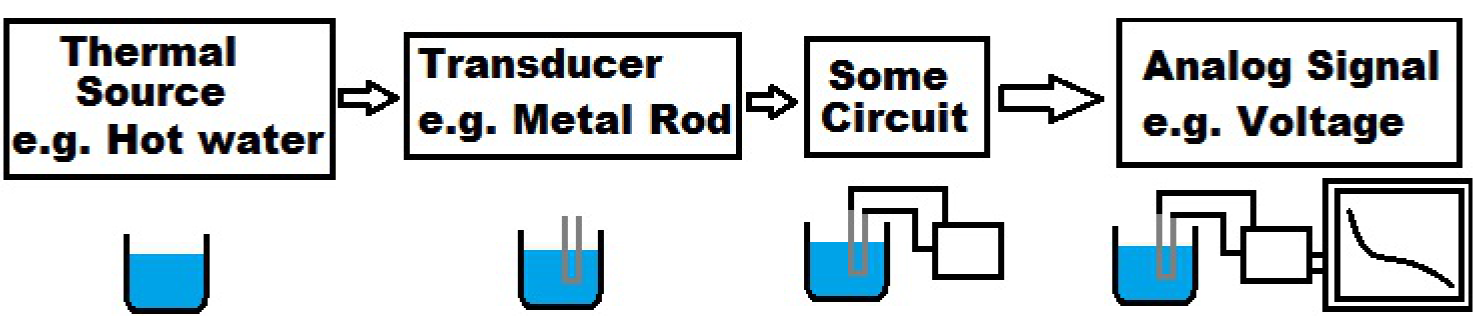
$$R(t) = R_0(1 + \alpha T + \beta T^2 + \lambda T^3 + ...)$$

But β is 10000 times smaller than α , λ is 100000 times smaller than β, so actually the first equation is already a good approximation of the second equation.

The more important factor is that, the first eqaution is linear.

Then, by using electrical circuit techniques (such as potential divider, Bridge), the change of resistance can be translated into change in voltage, and thus the temperature change measured can be converted into electrical signal.

The whole process is like this



There are lots of transducers (for temperature measurement, the other transducer are thermocuples, Infra-red pyrometer, thermistor, PN-Junction), thus it is impossible to discuss all the transducer here, the metal rod example above just act as a general idea of how to use transducer to collect data from physical world.

2.2 Amplifcation and LPF

The signal extracted from the transducer may be too small or too large. Thus amplifcation is needed.

In same logic, the signal extracted from the transducer may have very high frequency component that is not suitable for the sampler to get sample from it. Thus a low pass filter is added to adjust the bandwidth of the signal by decaying high frequency component.

Mathematically, for an input signal $x(t)$ having the following form

$$x(t) = a_0 + a_1 \cos \omega_1 t + a_2 \cos \omega_2 t + ... + a_n \cos \omega_n t + ...$$

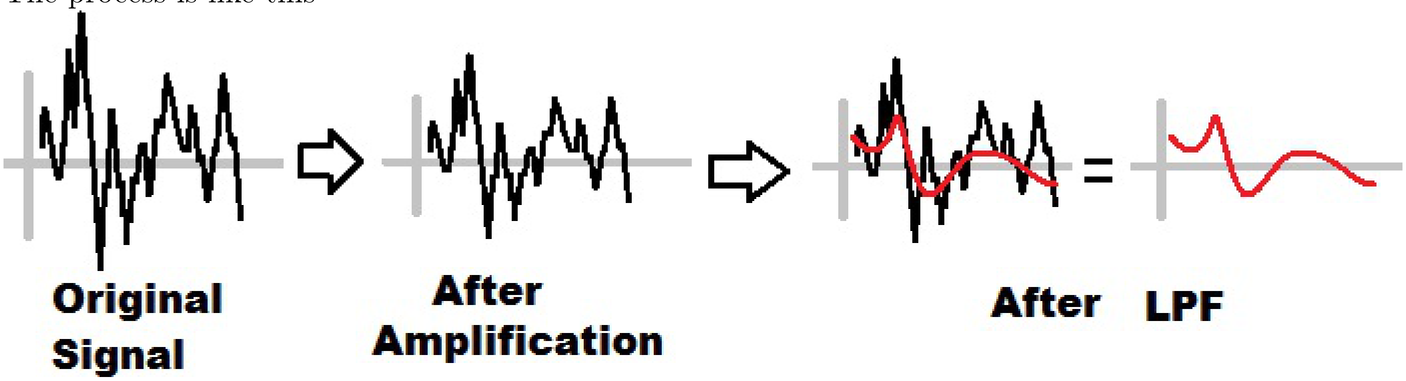
After amplification with factor K , the signal becomes

$$x'(t) = Kx(t) = a_0 K + a_1 K \cos \omega_1 t + a_2 K \cos \omega_2 t + ... + a_n K \cos \omega_n t + ...$$

And after passing through the low pass filter with cutoff frequency ω_c , the signal becomes

$$x"(t) = a_0 K + a_1 K \cos \omega_1 t + a_2 K \cos \omega_2 t + ... + a_m K \cos \omega_m t \qquad s.t. \, \omega_{m+1} > \omega_c$$

The process is like this



An example for illustration : A signal is in the form $x(t) = 0.4 + 1.2 \cos 300\pi t + 0.8 \cos 600\pi t - \cos 2400\pi t$

After amplification with a factor of 4 : $x'(t) = 1.6 + 4.8 \cos 300\pi t + 3.2 \cos 600\pi t - 4 \cos 2400\pi t$
After filtering with cutoff frequency $\omega_c = 1000\pi$: $x"(t) = 1.6 + 4.8 \cos 300\pi t + 3.2600\pi t$

For physically how to perform such amplification, filtering, please refer to hardware electronics.

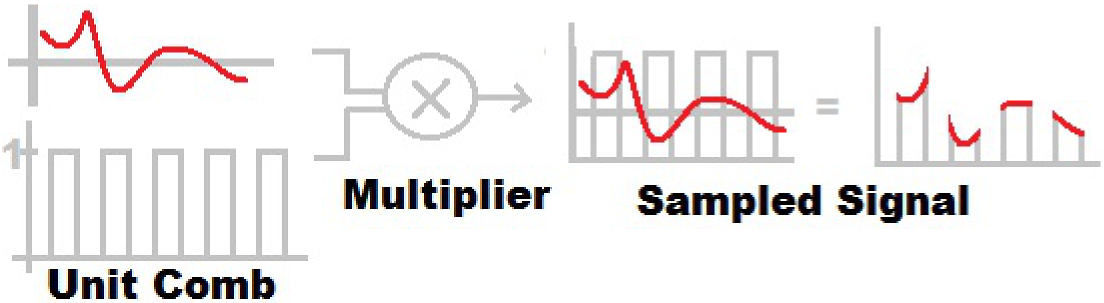
2.3 Sampling

Shannon-Nyquist Sampling Theorem

For a signal being sampled using frequency f_s , the signal being sampled should no component that having frequency high than $\frac{f_s}{2}$. Otherwise, the signal can not be reconstructed without information loss.

Since the signal after amplification and filtering, is also a continuous wave , it is impossible to store all the points into the computer (even for a straight line , there are basically ∞ points, it requires ∞ space for storage !). Sampling is carried out to get some samples that , no information will loss and reduce the space needed to store the information.

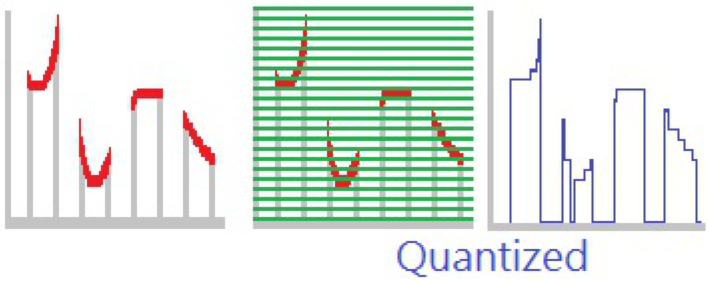
The most simple sampler is a sample and hold method, it is just a multiplier and impulse comb generator.



For physically how to perform such sampling, multplication, please refer to hardware electronics.

2.4 Quantization

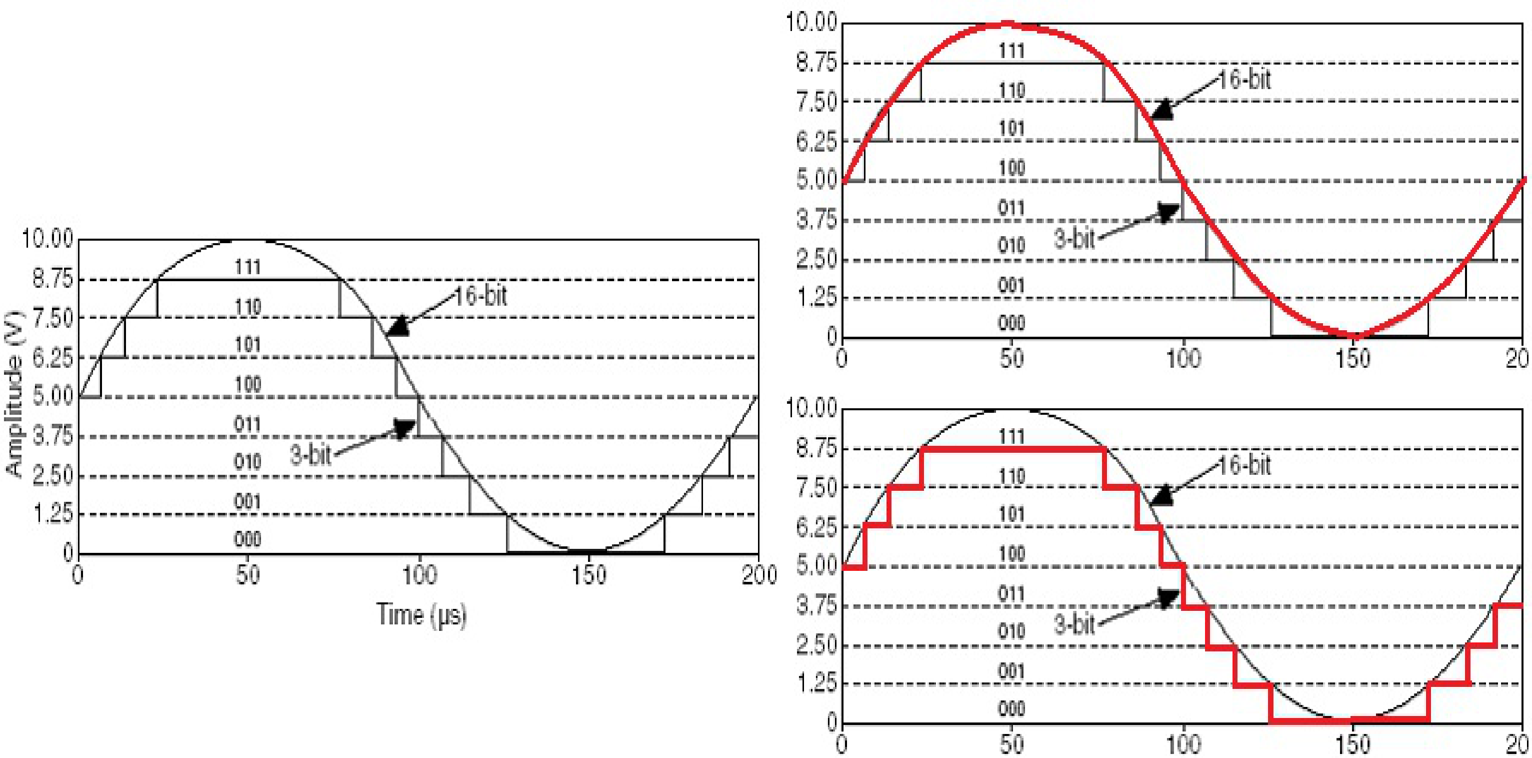
After the sampling the signal need to convert to digital form via a quantizer.



End use a encoder to encode the value into bits.

Quantization Error

A finite number of value can not represent infinite amount of value. Thus the difference between real analog signal and the digital signal causing by quantization is the quantization error.



Thus the quantization error can be found by the following

For a signal $x(t)$ and it's quantized version $x_q(t)$, suppose the quantization is carried out by M equal intervals, each interval with step side h.

In the diagram above, it has 8 interval with step size 0.05V

Let $f(m)$ be the propability that the signal $x(t)$ having magnitude in the m^{th} interval
i.e. $f(2)$ is the propability that $x(t)$ have magnitude at 2^{th} interval, that is, $2h$
i.e. $f(6)$ is the propability that $x(t)$ have magnitude at 6^{th} interval, $6h$

Then $f(m)dm$ is the propability that signal $x(t)$ having magnitude in the $m + \frac{h}{2}$ to $m - \frac{h}{2}$

Thus the mean square quantization error is

$$\overline{\epsilon}^2 = \int_{m_1 - \frac{h}{2}}^{m_1 + \frac{h}{2}} f(m) \left(m - m_1\right)^2 dm + \int_{m_2 - \frac{h}{2}}^{m_2 + \frac{h}{2}} f(m) \left(m - m_2\right)^2 dm + ...$$

Which is

$$\overline{\epsilon}^2 = \sum_{k=1}^{k=M} \int_{m_k - \frac{h}{2}}^{m_k + \frac{h}{2}} f(m) \left(m - m_k\right)^2 dm$$

Since $m - m_k = dm$, denote it as x , and assume $f(m)$ is same for all interval, denote it as f

$$\overline{\epsilon}^2 = \sum_{k=1}^{k=M} f \int_{-\frac{h}{2}}^{+\frac{h}{2}} x^2 dx$$

$$\overline{\epsilon}^2 = \frac{h^3}{12} \sum_{k=1}^{k=M} f = \frac{h^2}{12} \sum_{k=1}^{k=M} fh$$

Assume the propability distribution is linear, $f = \dfrac{1}{Mh}$

$$\overline{\epsilon}^2 = \frac{h^2}{12} \sum_{k=1}^{k=M} \frac{1}{M} = \frac{h^2}{12}$$

Since $h = \dfrac{V_{p-p}}{M}$, where V_{p-p} is the peak-to-peak voltage value

$$\overline{\epsilon}^2 = \frac{1}{12} \left(\frac{V}{M}\right)^2$$

Thus, this is the noise power

For a sine wave having amplitue $\dfrac{V_{p-p}}{2}$, it's RMS value is $\dfrac{V_{p-p}}{2\sqrt{2}}$, and power is $\dfrac{V_{p-p}^2}{8}$

Thus, the SNR (Signal to Noise Ratio)is

$$SNR_{Quantization} = \frac{S}{N} = \frac{\frac{V^2}{8}}{\frac{1}{12}\frac{V^2}{M^2}} = \frac{3}{2} M^2$$

Intuitively, it makes sense, since the more step you use (smaller step size), the more accurate the approximation is, thus the signal to noise ratio should increase!

2.5 Encoding

After the quantization, the signal becomes a series of integer values. Then an encoder can encode these values into bits.

For example, a 3 bit system, having the input as : 0 0 3 2 0 1 1 1 4
Then the output will be 000 000 011 010 000 001 001 001 110

For physically how to perform such encoding, please refer to computer systems.

3 Conclusion

After these steps,the signal can now fire into the DSP (Digital Signal Processor) to perform certain process to get useful information.

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