Responses to some question from students

Ch4 Q17c. Convolution Integral.

$$h(t) = e^{-2t}u(t) \quad , \quad x(t) = e^{-t}u(-t)$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad \Leftarrow \text{definition.}$$

Step-by-step illustration that even monkey can understand

① Since $x(t) = e^{-t}u(-t)$

Therefore $x(\tau) = e^{-\tau}u(-\tau)$, just directly put $t=\tau$

② Since $h(t) = e^{-2t}u(t)$

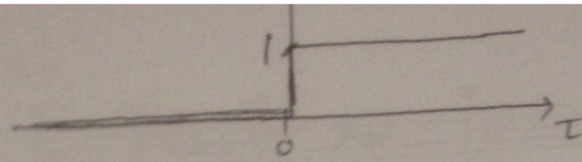
Thus $h(t-\tau) = e^{-2(t-\tau)}u(t-\tau)$, again, replace t in $h(\ )$ by $t-\tau$ directly.

③. $x(\tau)\,h(t-\tau)$ is thus

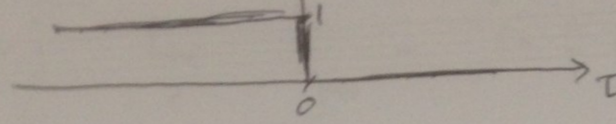
$$x(\tau)\,h(t-\tau) = e^{-2(t-\tau)}e^{-\tau}\,u(t-\tau)\,u(-\tau)$$

The "hard part" is "$u(t-\tau)\,u(-\tau)$"

④ $u(\tau)$

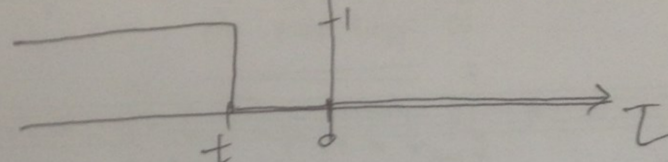


So $u(-\tau)$

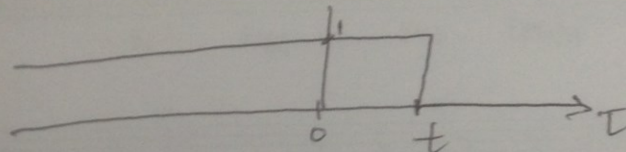


For $u(t-\tau)$, the "t" just shift $u(-\tau)$ ← or →

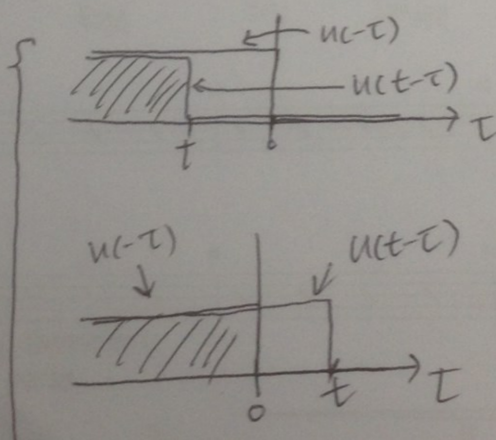
if t is negative, then $u(t-\tau)$ looks like



if t is positive, then $u(t-\tau)$ looks like



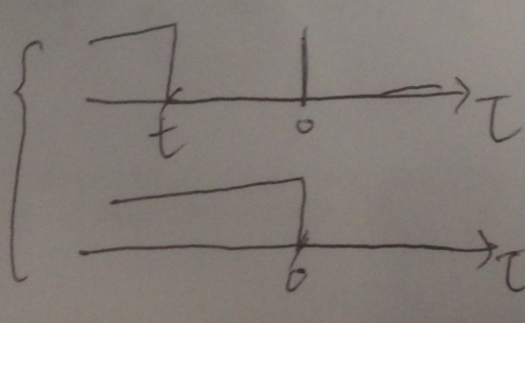
Thus $u(-\tau)\,u(t-\tau) = \Bigg\{$



$t<0$

$t>0$

$= \Bigg\{$



$t<0$

$t>0$

③ Therefore.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \begin{cases} \int_{-\infty}^{t} e^{-2(t-\tau)} e^{-\tau}\, d\tau, & t<0 \\ \int_{-\infty}^{0} e^{-2(t-\tau)} e^{-\tau}\, d\tau, & t>0 \end{cases}$$

and so on.

Ch4 17b)

$$\begin{cases} h(t) = e^{-2t} u(t) \\ x(t) = u(t+1) \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

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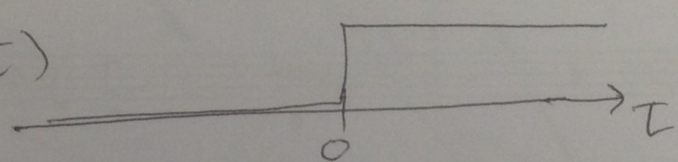
① $x(\tau) = u(\tau+1)$

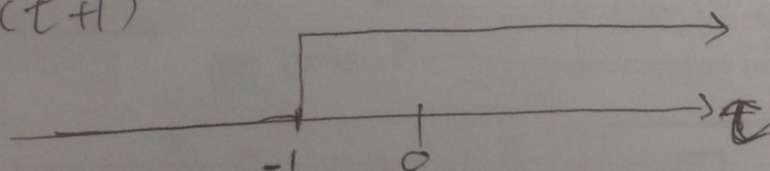
② $h(t-\tau) = e^{-2(t-\tau)} u(t-\tau)$

③ $x(\tau)\,h(t-\tau) = e^{-2(t-\tau)} u(\tau+1)\,u(t-\tau)$

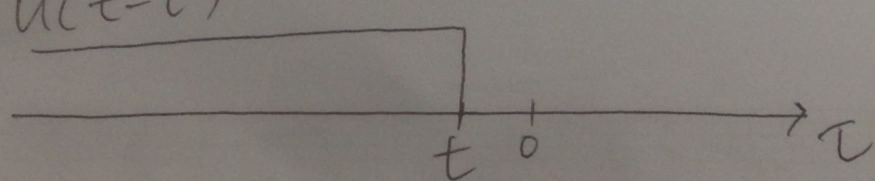
④ $u(\tau)$



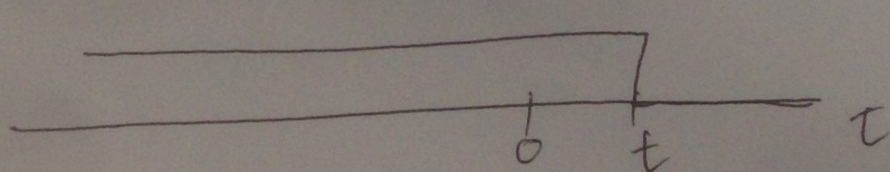
$u(\tau+1)$



$u(t-\tau)$

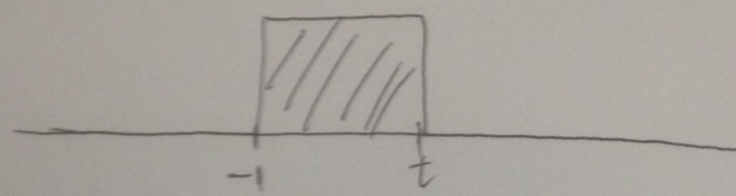


$t<0$



$t>0$

$u(t-\tau)\, u(\tau+1)$



for any t.

$$\therefore\ y(t) = \int_{-1}^{t} e^{-2(t-\tau)}\, d\tau$$

and so on !

ch4

6.18 a).

Notes

$x(t) = \sin t \; u(t)$

$h(t) = \sin t \; [u(t) - u(t-2\pi)]$

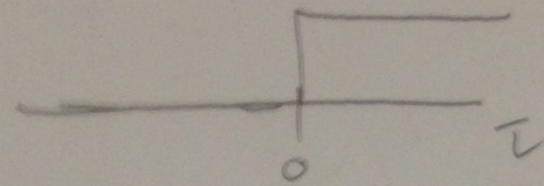
$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

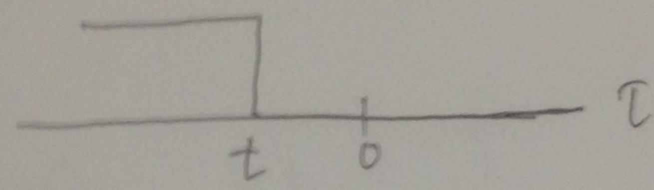
$= \int_{-\infty}^{\infty} \sin\tau \; u(\tau) \; \sin(t-\tau)\left[u(t-\tau) - u(t-\tau-2\pi)\right] d\tau$

$= \int_{-\infty}^{\infty} \sin\tau \; \sin(t-\tau) \; u(\tau)\left[u(t-\tau) - u(t-\tau-2\pi)\right] d\tau$

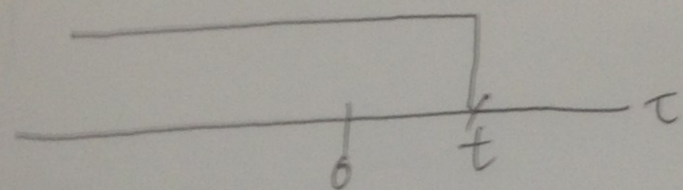
$u(\tau)$



$u(t-\tau)$

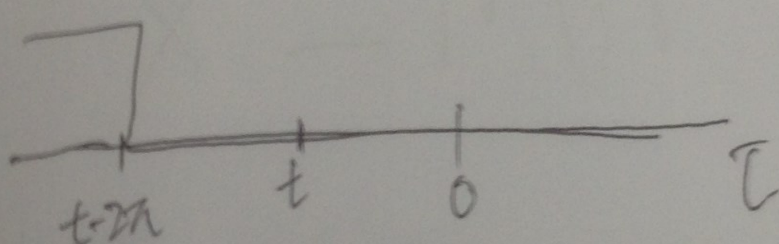


$t<0$

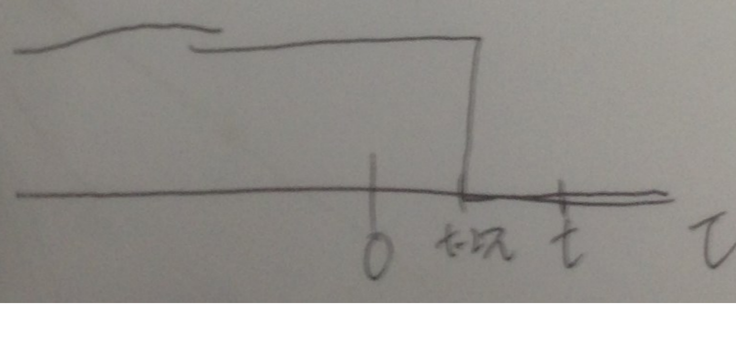


$t>0.$

$u(t-\tau-2\lambda)$

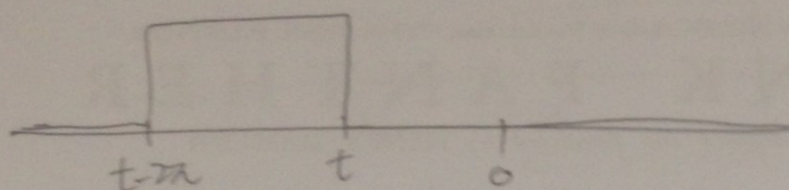


$t<0$

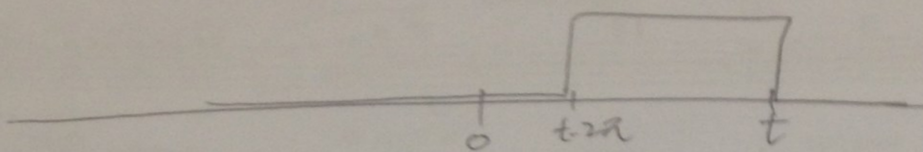


$t>0$

$u(t-\tau) - u(t-\tau-2\pi)$

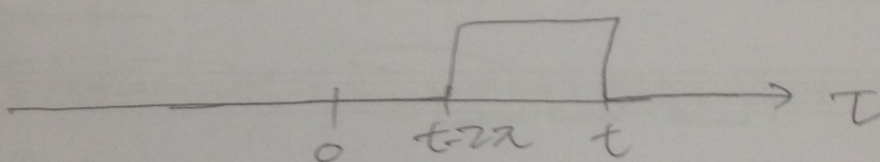


$t<0$



$t>0$

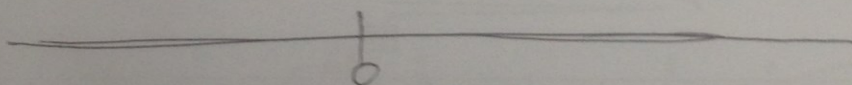
$u(\tau)\left[u(t-\tau) - u(t-\tau-2\pi)\right]$

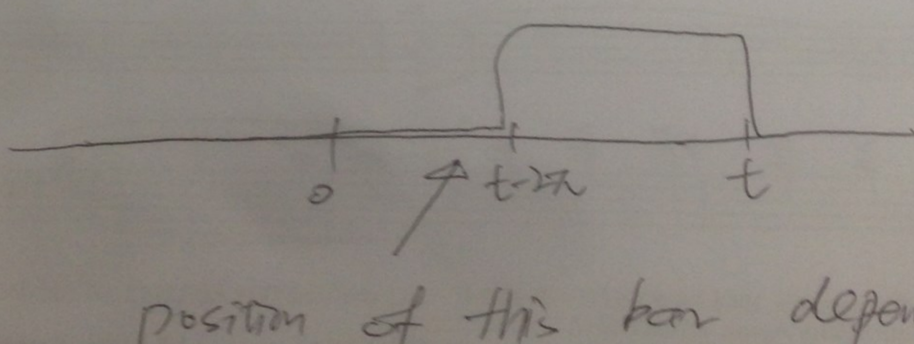


$t>0$

all zero.

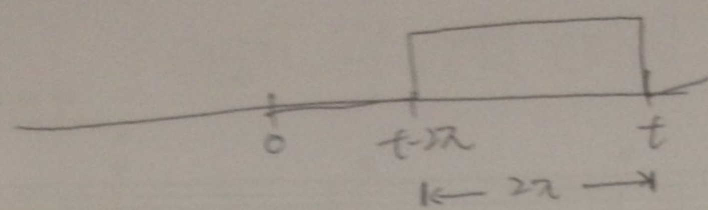
$t<0$



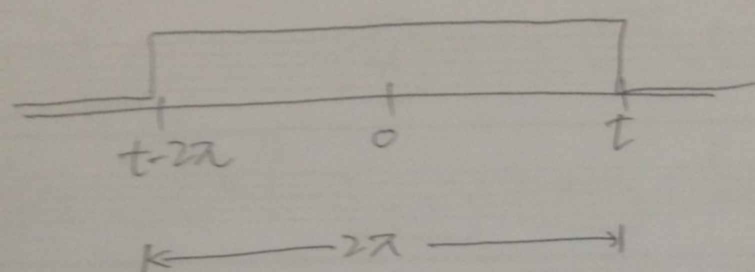


position of this bar depends on whether $t \lessgtr 2\pi$!

if $t > 2\lambda > 0$



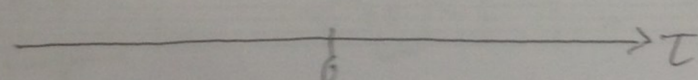
if $t < 2\lambda$, $t-2\lambda < 0$.



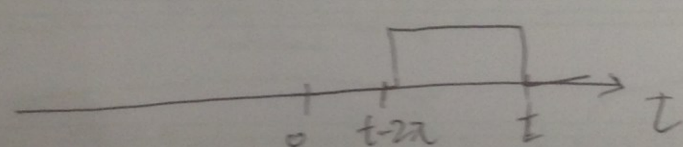
Therefore !

$u(\tau)\left[u(t-\tau) - u(t-\tau-2\lambda)\right]$

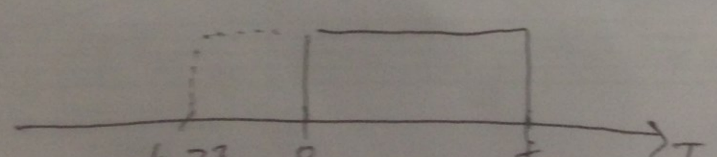
=



$t<0$



$t>2\lambda$



$0<t<2\lambda$

$$y(t) = \begin{cases} \int_0^0 \sin(\tau)\sin(t-\tau)\,d\tau = 0 & t<0 \\ \int_{t-2\pi}^{t} \sin(\tau)\sin(t-\tau)\,d\tau & t>2\pi \\ \int_0^t \sin(\tau)\sin(t-\tau)\,d\tau & 0<t<2\pi \end{cases}$$

and so on.

A student ask me the difference between Q10 and Q12b in Topic 3.

Topic 3 (ch12) Q10 & Q12(b)

$$h(n) = \{3, 2, 1\}$$

$$= 3\delta(n) + 2\delta(n-1) + 1\delta(n-2)$$

$g(n)$ is unit-step respond

$\therefore$ $g(n)$ is $y(n)$ when $x(n)$ is $u(n)$

$$g(n) = 3u(n) + 2u(n-1) + u(n-2)$$

$h(n)$ is impulse respond.

i.e. $h(n)$ is the $y(n)$ when $x(n)$ is $\delta(n)$

"$d(n)$ in book"

$\boxed{\text{Step-respond} \neq \text{impulse respond.}}$

A ~~critical~~ mistake in Tutorial 2.(6.3b)

$\cos(2\pi 4t)$

$f = 4$

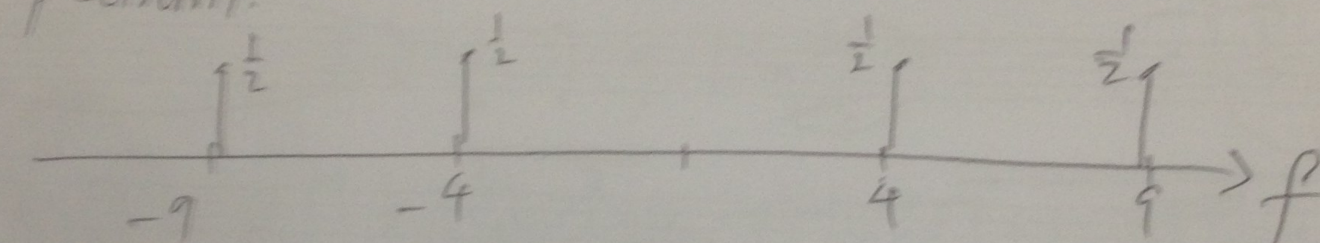
$T = \frac{1}{4}$

$\cos(3\pi 6t)$

$f = 9$

$T = \cancel{\frac{1}{18}}\ \frac{1}{9}$

spectrum.



GCF (HCF) of 4, 9 is 1.

So fundamental frequency is 1

So period is $\frac{1}{1} = 1$

NOT 36.

-END-