The mystery of a phasor calculation problem

2016-Oct-13

This document discusses an interesting thing in phasor calculation.

First of all, the following is a question I received from one student (anonymous):

Consider there are two sinusoids

$$x_1(t) = Re\{A_1 e^{j\phi_1} e^{j\omega t}\} = Re\{36 e^{-j1.97} e^{j2\pi\cdot 4000t}\}$$

$$x_2(t) = Re\{A_2 e^{j\phi_2} e^{j\omega t}\} = Re\{43.2 e^{j2.7} e^{j2\pi\cdot 4000t}\}$$

The third sinusoid, $x_3(t)$, can then be expressed as the sum

$$x_3(t) = Re\{(A_1 e^{j\phi_1} + A_2 e^{j\phi_2}) e^{j\omega t}\} = Re\{A_3 e^{j\phi_3} e^{j\omega t}\}$$

Substituting in values for A_1, A_2, ϕ_1, and ϕ_2, and solving for A_3 and ϕ_3 yields

$$x_3(t) = Re\{A_3 e^{j\phi_3} e^{j\omega t}\} = Re\{55.1 e^{-j2.87} e^{j\omega t}\}$$

Question: how to get the red circle value????

Question in English: Given a x_1 and x_2, find x_3 (using phasor). But I can't get the red value.

My comment: The student raise this question is probably due to the fact that the solution computed by hand is **different** from the solution computed by MATLAB.

Let’s look at the solution computed by hands

Normal	Phasor
$x_1 = A_1 e^{j\omega t} e^{j\phi_1}$	$\tilde{x}_1 = A_1 \angle \phi_1$
$x_2 = A_2 e^{j\omega t} e^{j\phi_2}$	$\tilde{x}_2 = A_2 \angle \phi_2$
$x_3 = A_3 e^{j\omega t} e^{j\phi_3}$	$\tilde{x}_3 = A_3 \angle \phi_3$
$x_3 = x_1 + x_2$	$\tilde{x}_3 = \tilde{x}_1 + \tilde{x}_2$
$\Leftrightarrow A_3 e^{j\omega t} e^{j\phi_3} = A_1 e^{j\omega t} e^{j\phi_1} + A_2 e^{j\omega t} e^{j\phi_2}$	$A_3 \angle \phi_3 = A_1 \angle \phi_1 + A_2 \angle \phi_2$

$$\begin{cases} A_1 = 36 \\ A_2 = 43.2 \\ \phi_1 = -1.97 \\ \phi_2 = 2.7 \end{cases}$$

$$\tilde{x}_1 = 36 \angle -1.97 = -13.9 - 33i$$

$$\tilde{x}_2 = 43.2 \angle 2.7 = -39 + 18i$$

$$\tilde{x}_3 = \tilde{x}_1 + \tilde{x}_2 = -53 - 14.7i$$

$$A_3 = \sqrt{re(\tilde{x}_3)^2 + imag(\tilde{x}_3)} = 55$$

$$\angle \phi_3 = \tan^{-1}\left[\frac{imag(\tilde{x}_3)}{re(\tilde{x}_3)}\right] = 0.27$$

* 0.27 (radian) ≒ 15.46° (degree)

$\phi_3 = 0.27$

What about $\phi_3' = \phi_3 + \pi$

$\phi_3'' = \phi_3 - 2\pi$

since

$\tan(\phi_3) = \tan(\phi_3 + \pi)$

$\tan(\phi_3) = \tan(\phi_3 - 2\pi)$

therefore ϕ_3', ϕ_3'', or $(\phi_3 - 2k\pi + h\pi)$

(k: any integer; h: 0 or 1)

are other solution to ϕ_3

- But Range of principal value of arc tangent is $-\frac{\pi}{2} < \theta < +\frac{\pi}{2}$

therefore $\phi_3 = 0.27$ is correct

$\phi_3' = 3.41$
$\phi_3'' = -2.87$
$\phi_3' - 2k\pi + h\pi$

} are "not so correct" !!!

That is, indeed the red circle one is NOT quite correct!!!!!

MATLAB code

```
clear all, close all,clc
A_1 = 36;
A_2 = 43.2;
phi_1 = -1.97;
phi_2 = 2.7;
x_1 = A_1 * exp( i* phi_1 );
x_2 = A_2 * exp( i* phi_2 );
x_3 = x_1 + x_2;
A_3 = abs(x_3);
phi_3 = angle(x_3);
A_3
phi_3
```

Surprisingly, the MATLAB solution is:

```
Command Window

A_3 =

   55.0494


phi_3 =

   -2.8712
```

It does not agree with what I said before!!

For phi_3, the solution by hand is 0.27, but MATLAB said it is -2.87!!

What's happening?

If you look at the MATLAB angle function documentation, it said

> **Algorithms**
>
> The `angle` function can be expressed as `angle(z) = imag(log(z)) = atan2(imag(z),real(z))`.

The key is the arctan2 function!

What is atan2 function?

The following document is one of my note typed few years ago, you can find the complete document at: http://www.eee.hku.hk/~msang/atan2.pdf

The atan2() function

February 6, 2013

The function $tan^{-1}\frac{y}{x}$ has two input parameter, y and x

When $x, y < 0$, for example, (-3,-1) , $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-1}{-1} = \tan^{-1}\frac{1}{3}$, it is same as the input $(+3, 1)$

Then the function can not differentiate $(\pm x, \pm y)$, the output is **not unique**

When tackling a problem that uniqueness of the solution is required (for example , inverse kinematics problem in robotics) , then a better function atan2()is used, the mathematical definition of atan2() is

$$\texttt{atan2(y,x)} = \begin{cases} \tan^{-1}\frac{y}{x} & x > 0 \\ \left(\tan^{-1}\frac{y}{x}\right) + \pi & x < 0, y \geq 0 \\ \left(\tan^{-1}\frac{y}{x}\right) - \pi & x < 0, y < 0 \\ +\frac{\pi}{2} & x = 0, y > 0 \\ -\frac{\pi}{2} & x = 0, y < 0 \\ \text{Undefined} & x = y = 0 \end{cases}$$

The atan2(y,x) also equals to

$$\texttt{atan2(y,x)} = \frac{2y}{\sqrt{x^2 + y^2} + x}$$

Final comments:

By definition of arc tangent, the solution to the phase angle of x_3 should be 0.27.The computer generated result is based on the use of "arc tangent 2", not "arc tangent", therefore it generates the solution -2.87. But based on the fact that arctangent value is ranged within $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so the correct solution should be 0.27 not -2.87 !!!

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