About AICE1004

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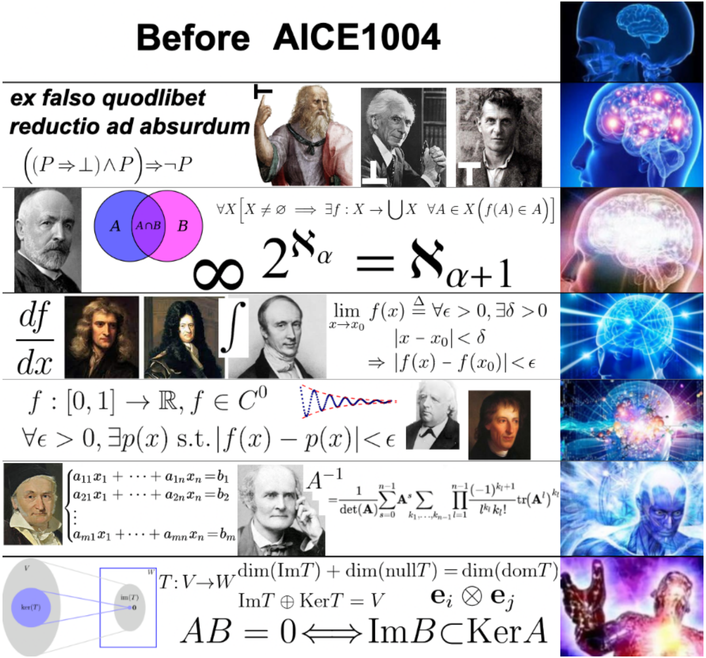
Hardest module this sem

- Math is hard
- Math is abstract
- Math needs memory
- Math needs understanding
- Math needs imagination
- Math needs practise

the most important module



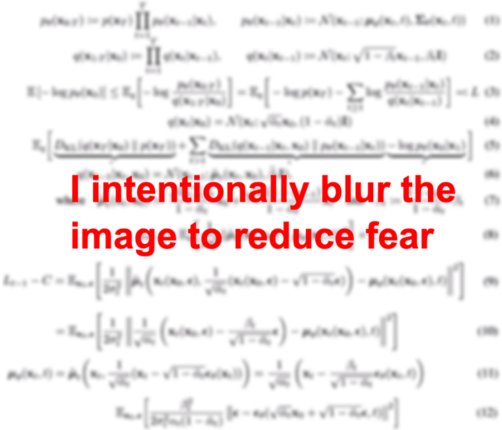
AI image generation

Input: “Students studying mathematics”

AI generate:

Why math is important

you need math to understand why AI works



Your instructors

- Andersen (Man Shun) Ang (Module leader)
- Website: angms.science
- Math geek
  Optimization Theorist
  Your final boss this semester

$$\left\{ v \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists x_k \to x, f(x_k) \to f(x) \\ v_k \in \partial f(x_k), \lim\limits_{k\to\infty} v_k = v \end{array} \right\}$$

- Office: Building 32 Room 3057

- Zhanxing Zhu
- Website: zhanxingzhu.github.io
- Badminton zealot
  Deep Learning Theorist
  DL uses lots of linear algebra $\implies$ teach linear algebra

$$\mathcal{L}\Big(N, D, T; f(x;\theta)\Big) = E + \frac{A}{N^\alpha} + \frac{B}{D^\beta} + \frac{C}{T^\gamma}$$
$$\text{s.t. } f(x;\theta) = f_1 \circ f_2 \circ \cdots \circ f_L(x)$$

- Office: Building 32 Room 3XXX

Syllabus

Part 1. math of "formality"
Logic & set

Proposition, truth, $\mathrm{Id}, \top, \bot, \neg$
$\wedge, \vee, \overline{\wedge}, \underline{\vee}, \forall, \exists, \implies, \iff$
$\vdash, \models$, proofs

$\{\}, \in, \notin, \subset, \cap, \cup, \setminus, \Delta, A^c, \varnothing,$
$2^S, |S|, \aleph, \infty,$
$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}, \mathbb{C}$
$\times$, R, [ ], / $\sim$

dom, codom, Img, 1→1, onto, 1-1

~~LEAN 4~~

Part 2. math of "change"
Calculus

Number, elementary functions, inequality

calculus: differentiation, integration

analysis (calculus at proof level):
sequence, definition of limit, continuity,
L'Hospital rule, Taylor series, Riemann sum

Wolfram Alpha (Lab 1 M1)

Part 3. math of "structure"
Linear algebra

Matrix-vector algebra, $\boldsymbol{Ax} = \boldsymbol{b}$, Gaussian elimination

inverse, eigendecomposition, characteristic equation, determinant

Abstract linear algebra: combination, span, column space, independence, orthogonality, rank, dimension, kernel, quotient

NumPy (Lab 2 M2)

AICE1004 Mathematics for Artificial Intelligence and Computer Engineering (I)

Time commitment: 150 hours

Lecture	32h
Independent study	108h
Laboratories	10h

Assessment (School rule: $\geq 40\%$ to pass)

Computing Laboratories (lab)	5%
Coursework (mid-term test)	35%
Exam	60%

Online platform: Moodle
https://moodle.ecs.soton.ac.uk/l

- Lecture notes
- Exercises
- Announcement
- Lab
- Ask question here so that everybody can see it

Lecture

- Monday 11:00-11:55, 27/2001
- Thursday 09:00-10:55, 35/1005
- Friday 10:00-11:55, 27/2001
- Map
  https://maps.southampton.ac.uk/

Useful websites for new students

- SUSSED https://sotonac.sharepoint.com/
- StudentHub https://www.southampton.ac.uk/studentservices/index.page
  - information
  - fees, finance, accommodation
  - wellbeing, disability, careers
  - course administration, change program
- School rule
  https://www.southampton.ac.uk/about/governance/regulations-policies/taught-students/general/results-ug-integrated-masters
  9.1 Mark Scheme: Pass $:= \geq 40$
- https://data.soton.ac.uk/
  - Map, Building
  - Which week it is , Term date, University Calendar
- https://timetable.soton.ac.uk
- https://moodle.ecs.soton.ac.uk/login

12-week *tentative* plan

w	Topic, by	Content	Assessment
1	Logic, A	Proposition, truth, Id, $\top$, $\bot$, $\neg$, $\wedge$, $\vee$, $\overline{\wedge}$, $\underline{\vee}$, $\forall$, $\exists$, $\Longrightarrow$, $\Longleftrightarrow$, $\vdash$, $\models$, proof	
2	Set, A	$\{\}$, $\in$, $\notin$, $\subset$, $\cap$, $\cup$, $\varnothing$, $\mathbb{N}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$, $\times$, R, $1\!\rightarrow\!1$, onto, 1-1, 2^A, $\lvert S\rvert$, $\aleph$	lab1 (2.5%)
3	Calculus, A	Number system, elementary functions,	
4	Calculus, A	Limit, continuity, derivative	
5	Calculus, A	Differentiation, Taylor series	
6	Calculus, A	antiderivative, integration, Riemann sum, Fundamental theorem of calculus	mid-term (35%)
7	LA, Z	Vector and matrix algebra, their geometric meaning	
8	LA, Z	Linear system, Gaussian Elimination, their geometric meaning	
9	LA, Z	Inverse, det, characteristic equation, Eigenvalue, eigenvector	
10	LA, Z	vector space: comb, span, Col, basis, dim, rank, ker, quotient	
11	Revision, A	revision of week 1 - week 6	lab2(2.5%)
12	Revision, Z	revision of week 7 - week 10	
			Exam (60%)

	October 2024					November 2024				December 2024				January 2025			
Mon		7	14	21	28	4	11	18	25	2	9	16	23		6	13	20
Tue	1	8	15	22	29	5	12	19	26	3	10	17	24		7	14	21
Wed	2	9	16	23	30	6	13	20	27	4	11	18	25	1	8	15	22
Thu	3	10	17	24	31	7	14	21	28	5	12	19	26	2	9	16	23
Fri	4	11	18	25	1	8	15	22	29	6	13	20	27	3	10	17	24
Sat	5	12	19	26	2	9	16	23	30	7	14	21	28	4	11	18	25
Sun	6	13	20	27	3	10	17	24	1	8	15	22	29	5	12	19	26
Semester Wks	1	2	3	4	5	6	7	8	9	10	11				12	Ex	Ex

Good news	Bad news
No homework (tons of exercises in MadBook)	Mid-term in Week 6 & Final exam (Mock midterm/exam will be provided)
You probably learned Calculus & Linear Algebra	We will look at them at abstract level & deeper level
So you are familiar with it	We do proof

Example exam question

Solve the following system of equations, using matrix notation, and the Gaussian elimination plus back-substitution method.

$$\begin{array}{rcrcrcr} x & + & y & - & z & = & 1 \\ 2x & + & y & - & z & = & 6 \\ 3x & + & 7y & - & 6z & = & -1 \end{array}$$

Suggested solution
Step 1. Construct the augmented matrix

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & -1 & 6 \\ 3 & 7 & -6 & -1 \end{bmatrix}$$

Step 2. $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & 4 \\ 0 & 4 & -3 & -4 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 4R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 1 & 12 \end{bmatrix}$$

Step 3. We have

$$\begin{array}{rcrcrcr} x & + & y & - & z & = & 1 \\ & - & y & + & z & = & 4 \\ & & & & z & = & 12 \end{array}$$

Using back-substitution gives $(x, y, z) = (5, 8, 12)$.

Example exam question

Given a matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$.

The eigenvalues of $\boldsymbol{A}$ are (there are more than one option that is correct, you get one point by picking all the correct options, you get zero point otherwise):

(a) 2
(b) -2
(c) 5
(d) -5

Solution: (a) and (d)

$$
\begin{aligned}
& \det(\boldsymbol{A} - \lambda \boldsymbol{I}) = 0 \\
\iff & \begin{vmatrix} 1-\lambda & 2 \\ 3 & -4-\lambda \end{vmatrix} = 0 \\
\iff & (1-\lambda)(-4-\lambda) - 2 \cdot 3 = 0 \\
\iff & (\lambda - 1)(\lambda + 4) - 6 = 0 \\
\iff & \lambda^2 + 3\lambda - 10 = 0 \\
\iff & (\lambda - 2)(\lambda + 5) = 0 \\
\iff & \lambda \in \{+2, -5\}
\end{aligned}
$$

Marking:

- Pick (a) and (d) 1 point
- Otherwise 0 point

Assessment (School rule: $> 40\%$ to pass)

- L_1: result of lab1 — 2.5%
- L_2: result of lab2 — 2.5%
- T: mid-term test — 35%
- E: exam — 60%
- G: grade
- Example: appear in all the labs, test and exam — $G = 0.025L_1 + 0.025L_2 + 0.35T + 0.6E.$
- Example: missed lab1 — $G = 0.025L_2 + 0.35T + 0.625E$
- Example: missed test — $G = 0.025L_1 + 0.025L_2 + 0.95E$
- Example: missed lab1 and test — $G = 0.025L_2 + 0.975E$
- Example: missed lab1, lab2 and test — $G = E$
- Example: missed exam — $G = 0.025L_1 + 0.025L_2 + 0.35T$
- Fun fact: you can actually use one equation to express all the situations above

~~Lab M0 Logic: LEAN 4.0~~

Lab M1 Calculus: Wolfam Alpha (Mathematica)

Lab M2 Linear Algebra: Python (NumPy) / MATLAB

Readings: Lecture notes and MadBookPro are sufficient

- Make your own notes!
- Want more?
  - Discrete mathematics and its applications by Kenneth H. Rosen — QA39 ROS
    Logic and Proofs, Sets, Functions, Matrices
  - Calculus : a complete course (10th ed) by Robert Adams & Christopher Essex — QA303 ADA
    Chapter 1,2,5,6,10
  - Calculus (3rd ed) by Michael Spivak — QA303 SPI
    First 400 pages
  - Schaum's outlines calculus — QA303 AYR, library has ebook
    Chapter 1-35 or the whole book
  - Schaum's outlines, advanced calculus — QA303 SPI, library has ebook
    Chapter 3,4,5,7
  - Introduction to Applied Linear Algebra - Vectors, Matrices, and Least Squares by Stephen Boyd and Lieven Vandenberghe
    Free pdf https://web.stanford.edu/~boyd/vmls/vmls.pdf
    Slide by Boyd & Vandenberghe https://web.stanford.edu/~boyd/vmls/vmls-slides.pdf
  - Introduction to linear algebra by Gilbert Strang — QA184.2 STR
    Solving linear equations, Vector spaces and subspaces, Orthogonality, Determinants, Eigenvalues and eigenvectors
  - Schaum's outlines, Linear algebra — QA251 LIP, library has ebook
    Chapter 1,2,3,4,8

AICE1004
(Academic Year 2024/25)

View Online

Reading for AICE1004

7 items

Discrete mathematics and its applications - Kenneth H. Rosen, 2019
Book | Additional

Schaum's Outline of Discrete Mathematics (3rd Edition) - Seymour Lipschutz, Marc Lipson, 2007
Book | Additional

Calculus : a complete course - Robert A. Adams, Christopher Essex, 2022
Book | Additional

Calculus - Frank Ayres, Elliott Mendelson, ©2013
Book | Additional

Advanced calculus - Robert C. Wrede, Murray R. Spiegel, ©2010
Book | Additional

Introduction to linear algebra - Gilbert Strang, 2016
Book | Additional

Linear algebra - Seymour Lipschutz, Marc Lipson, 2013
Book | Additional

YouTube

- Essence of calculus by 3Blue1Brown
  https://www.youtube.com/playlist?list=PLZHQObOWTQDMsr9K-rj53DwVRMYO3t5Yr
- Essence of linear algebra by 3Blue1Brown
  https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab
- Video 1 - 7 in Vectors for multi-variable calculus by Michael Penn
  https://www.youtube.com/playlist?list=PL22w63XsKjqyC3bWVd5EjGEVN9pqq55yF
- Techniques of Integration by Michael Penn
  https://www.youtube.com/playlist?list=PL22w63XsKjqy_Hgl0PAhSNsTCx84UpARt

The remaining slides are used to warn you that math is hard

HARDEST module this sem

- Math is hard
- Math is abstract
- Math needs memory
- Math needs understanding
- Math needs imagination
- Math needs practise

the most important module in AI

In lecturer
Your eye/ear: oh I understand!
Your brain/hand: no you don't!

Watch somebody doing math $\neq$ learning

you do math $\neq$ learning

(you do math) $\wedge \Big($(get it correct) $\vee$ (get it wrong but understand why wrong)$\Big) =$ learning

Warning: Math needs memory

Math is like $A \to B \to C \to D \to \ldots$

If you forgot A, you can't proceed to $B, C, \ldots$

Example.

Find the eigenvector of $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

you first need to know what are *vectors*, *eigenvalues*, so on

Warning: Math needs imagination

Math is abstract, pure memory is useless

you don't know what is it if you can't image a picture

Example.

$\det \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

$=$ the area of parallelogram of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

Warning: Math needs logical thinking

Math is structural: knowing a bunch of unstructured information is useless

If you can't form a "logic chain" in your head, you will fail

Example. Scaling a row by a constant multiplies the determinant by that constant.

If we scale the first row of $\begin{bmatrix} -4 & 3 & 3 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix}$, then $\det \begin{bmatrix} -8 & 6 & 6 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix} = 2 \det \begin{bmatrix} -4 & 3 & 3 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix}$.

$$\det \begin{bmatrix} -8 & 6 & 6 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix} = \det \left(\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 3 & 3 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix} \right)$$

$$= \det \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \det \begin{bmatrix} -4 & 3 & 3 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix}$$

based on $\det(\boldsymbol{AB}) = (\det \boldsymbol{A})(\det \boldsymbol{B})$

$$= 2 \det \begin{bmatrix} -4 & 3 & 3 \\ 8 & 7 & 3 \\ 4 & 3 & 3 \end{bmatrix}$$

det of diagonal matrix is the product of diagonal

High school / A-level / IB / GCSE / blablabla math are nothing

Bad math in high school:

No worry, work hard will be ok

Good math in high school:

Nothing special, you are only δ-better

$\{$bad high-school math$\} = 50$
$\{$good high-school math$\} = 85$
$\{$math$\} = 400000000000000000000000$[1]
$\{$good high-school math$\}$ is only $0.000000000000000000008\%$ better

Me? May be 9000 $\ll 400000000000000000000000$

[1] this number is the number of vertices of a "math knowledge graph"

In math you don't understand things, you just get used to them — von Neumann

See a new concept, **you don't feel "what's going on" until you do a tons of examples**

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \implies \boldsymbol{A} \oplus \boldsymbol{B} = \begin{bmatrix} a & b & & \\ c & d & & \\ & & e & f \\ & & g & h \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a & b \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} e \\ f \end{bmatrix} \implies \boldsymbol{A} \oplus \boldsymbol{B} = \begin{bmatrix} a & b & \\ & & e \\ & & f \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a & b \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} e & f \end{bmatrix} \implies \boldsymbol{A} \oplus \boldsymbol{B} = \begin{bmatrix} a & b & e & f \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a \\ b \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} e \\ f \end{bmatrix} \implies \boldsymbol{A} \oplus \boldsymbol{B} = \begin{bmatrix} a \\ b \\ e \\ f \end{bmatrix}$$

von Neumann is know as the "father of computer", but he is also a math god: C*-algebra, von Neumann mutual information, von Neumann decomposition, von Neumann spectral theorem

Math is crazy

- Cantor's diagonal prove of ∞
- Godel's completeness Theorem
- Godel-Maltsev Compactness Theorem of 1st-order logic
- Zermelo-Fraenkel set theory and Axiom of Choice
- Zorn's Lemma of partially ordered set
- Axioms of integer and Axioms of real number
- Bolzano-Weierstrass Theorem
- Robinson's Transfer Principle and Hyper-reals
- Weierstrass Approximation Theorem of continuous function
- Fermat's rule, Rolle's Theorem, Lagrange Theorem, Cauchy Theorem
- Fundamental Theorem of calculus
- Rank-null Theorem
- Fundamental Theorem of Linear Algebra
- Fundamental Homomorphism Theorem of vector space

Math is crazy (things we may talk about in the course)

- Not everything can be a set, for example $R \coloneqq \{x \ : \ x \in x\}$
- There are more real numbers in $[0, 1]$ than all integers
- There are things being "too large" to be a set, for example $C = \{\aleph_0, \aleph_1, \aleph_2, ...\}$
- The sum of all real number in the interval $[0, 1]$ is impossible to compute
- Some function is impossible to differentiate
- $\dfrac{\mathrm{d}^{0.5}}{\mathrm{d}^{0.5}x}x =$????
- $\displaystyle\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$ is wrong.
- Some function is impossible to integrate, for example $\displaystyle\int_0^1 \sin\left(\frac{1}{x}\right)\mathrm{d}x$
- All vector space has a basis

Example: proving converge of sequence using logic

A sequence $\{s_n\} \in \mathbb{R}$ *converges* to $L \in \mathbb{R}$ if and only if

$$(\forall \epsilon > 0)(\exists K \in \mathbb{N})\Big(\forall n \geq K \implies |s_n - L| < \epsilon\Big)$$

Now prove $\lim_{n \to \infty} \frac{1}{n} = 0$ using logic

Prove $0.99999999... = 1$

Example: calculus

You may think calculus is easy: $\int x^5 \mathrm{d}x = \frac{1}{6}x^6 + C$

How about: let a sequence $f_n(x) = \cos(nx)$

$f_1(x) = \cos x,\ f_2(x) = \cos(2x),\ f_3(x) = \cos(3x), ...$

Now,

$$\int \lim_{n \to \infty} f_n(x)\mathrm{d}x \overset{?}{=} \lim_{n \to \infty} \int f_n(x)\mathrm{d}x$$

- Does left hand side exists?
- Does right hand side exists?
- Does the equality holds?
- Why is it true?
- Why is it not true?
- What if $\int f^{-1}(x)\mathrm{d}x$?

Example: abstract linear algebra (You will learn this)

Invertible matrix theorem If $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, then following are all equivalent (either all true or all false for $\boldsymbol{A}$)

	1	$\boldsymbol{A}$ is non-singular $\iff$ $\boldsymbol{A}^{-1}$ exists
$\iff$	$2a$	$\boldsymbol{A}$ has left inverse under matrix multiplication $\iff$ there exists $\boldsymbol{B}$ such that $\boldsymbol{BA} = \boldsymbol{I}$
$\iff$	$2b$	$\boldsymbol{A}$ has right inverse under matrix multiplication $\iff$ there exists $\boldsymbol{C}$ such that $\boldsymbol{AC} = \boldsymbol{I}$
$\iff$	$2c$	$\boldsymbol{A}$ has inverse under matrix multiplication $\iff$ there exists $\boldsymbol{D}$ such that $\boldsymbol{DA} = \boldsymbol{AD} = \boldsymbol{I}$
$\iff$	$3a$	the reduced row-echelon form of $\boldsymbol{A}$ is $\boldsymbol{I}_n$
$\iff$	$3b$	the reduced column-echelon form of $\boldsymbol{A}$ is $\boldsymbol{I}_n$
$\iff$	$3c$	$\boldsymbol{A}$ has n pivots
$\iff$	$3d$	$\boldsymbol{A}$ can be expressed as a product of elementary matrices $\iff$ $\boldsymbol{A} = \boldsymbol{E}_1 \boldsymbol{E}_2 \cdots$
$\iff$	$4a$	$\boldsymbol{A} : \boldsymbol{x} \mapsto \boldsymbol{Ax}$ is surjective $\iff$ there exists at least one $\boldsymbol{x}$ that solves $\boldsymbol{Ax} = \boldsymbol{b}$
$\iff$	$4b$	$\boldsymbol{A} : \boldsymbol{x} \mapsto \boldsymbol{Ax}$ is injective $\iff$ there exists at most one $\boldsymbol{x}$ that solves $\boldsymbol{Ax} = \boldsymbol{b}$
$\iff$	$4c$	$\boldsymbol{A} : \boldsymbol{x} \mapsto \boldsymbol{Ax}$ is bijective $\iff$ there exists exactly one $\boldsymbol{x}$ that solves $\boldsymbol{Ax} = \boldsymbol{b}$
$\iff$	$4d$	$\boldsymbol{Ax} = \boldsymbol{b}$ is consistent for all $\boldsymbol{b} \in \mathbb{R}^n$
$\iff$	$5a$	columns of $\boldsymbol{A}$ are linearly independent
$\iff$	$5b$	rows of $\boldsymbol{A}$ are linearly independent
$\iff$	$5c$	columns of $\boldsymbol{A}$ span $\mathbb{R}^n$
$\iff$	$5d$	rows of $\boldsymbol{A}$ span $\mathbb{R}^n$
$\iff$	$5e$	columns of $\boldsymbol{A}$ form a basis for $\mathbb{R}^n$
$\iff$	$5f$	rows of $\boldsymbol{A}$ form a basis for $\mathbb{R}^n$
$\iff$	$6a$	$\boldsymbol{A}$ has rank n
$\iff$	$6b$	$\boldsymbol{A}$ has nullity 0
$\iff$	$6c$	$\boldsymbol{Ax} = \boldsymbol{0}$ has only the trivial solution $\boldsymbol{x} = \boldsymbol{0}$ $\iff$ $\ker \boldsymbol{A} = \{\boldsymbol{0}_n\}$
$\iff$	$6d$	the orthogonal complement of the null space of $\boldsymbol{A}$ is $\mathbb{R}^n$
$\iff$	$6e$	the orthogonal complement of the row space of $\boldsymbol{A}$ is $\boldsymbol{0}_n$
$\iff$	$6f$	the range of $\boldsymbol{A}$ is $\mathbb{R}^n$
$\iff$	$7a$	$\det(\boldsymbol{A}) \neq 0$
$\iff$	$7b$	the eigenvalues of $\boldsymbol{A}$ contains no zero $\iff$ 0 is not an eigenvalue of $\boldsymbol{A}$
$\iff$	$8a$	$\boldsymbol{A}^\top$ is invertible
$\iff$	$8b$	$\boldsymbol{A}^\top \boldsymbol{A}$ is invertible

If you want even more exercises

- (3193) Problems in Mathematical Analysis
  Boris Demidovich
- 3000 Solved Problems in Calculus
  Elliot Mendelson
- 3000 Solved Problems in Linear Algebra
  Lipschutz Seymour
- **How to study mathematics**
  Pattern Recognition
  Math is unlike other subject, all problems are unique, problem will not repeat, this is why memorization is useless in math

1466. $\int \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}+x\right)dx.$

1467. $\int \tan^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx.$

1468. $\int \frac{dx}{2\sin x+3\cos x-5}.$

1469. $\int \frac{dx}{2+3\cos^2 x}.$

1470. $\int \frac{dx}{\cos^2 x+2\sin x\cos x+2\sin^2 x}.$

1471. $\int \frac{dx}{\sin x\sin 2x}.$

1472. $\int \frac{dx}{(2+\cos x)(3+\cos x)}.$

1473. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4\tan x+1}}dx.$

1474. $\int \frac{\cos ax}{\sqrt{a^2+\sin^2 ax}}dx.$

1475. $\int \frac{x\,dx}{\cos^2 3x}.$

1476. $\int x\sin^2 x\,dx.$

1477. $\int x^2 e^{x^3}dx.$

1478. $\int xe^{2x}dx.$

1479. $\int x^2\ln\sqrt{1-x}\,dx.$

1480. $\int \frac{x\arctan x}{\sqrt{1+x^2}}dx.$

1481. $\int \sin^2\frac{x}{2}\cos\frac{3x}{2}dx.$

1482. $\int \frac{dx}{(\sin x+\cos x)^2}.$

1483. $\int \frac{dx}{(\tan x+1)\sin^2 x}.$

1484. $\int \sinh x\cosh x\,dx.$

1485. $\int \frac{\sinh\sqrt{1-x}}{\sqrt{1-x}}dx.$

1486. $\int \frac{\sinh x\cosh x}{\sinh^2 x+\cosh^2 x}dx.$

1487. $\int \frac{x}{\sinh^2 x}dx.$

1488. $\int \frac{dx}{e^{2x}-2e^x}.$

1489. $\int \frac{e^x}{e^{2x}-6e^x+13}dx.$

1490. $\int \frac{e^{2x}}{(e^x+1)^{\frac{1}{4}}}dx.$

1491. $\int \frac{2^x}{1-4^x}dx.$

1492. $\int (x^2-1)\,10^{-2x}dx.$

1493. $\int \sqrt{e^x+1}\,dx.$

1494. $\int \frac{\arctan x}{x^2}dx.$

1495. $\int x^3\arcsin\frac{1}{x}dx.$

1496. $\int \cos(\ln x)\,dx.$

1497. $\int (x^2-3x)\sin 5x\,dx.$

1498. $\int x\arctan(2x+3)\,dx.$

1499. $\int \arcsin\sqrt{x}\,dx.$

1500. $\int |x|\,dx.$

What comes after AICE1004?

- Vector calculus (AICE1008 Math II)
  $\subset$ Linear algebra $\oplus$ Calculus
- Scientific Computing (AICE2001)
  $\approx$ Linear Algebra II
- Optimization (COMP6260)
  queen of applied math
- Deep Learning
  $\approx$ Optimization $\oplus$ Statistics
  queen of machine learning(?)

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_3} \end{bmatrix}, \ \oint, \iint, \ (\nabla A) \cdot \boldsymbol{F}$$

$$G = (Df)^\top Df = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_2} \end{bmatrix}$$

$$d\boldsymbol{u} \otimes d\boldsymbol{v}, \quad d\boldsymbol{u} \wedge d\boldsymbol{v}$$

How to study math

- Bring pen, papers & computer,
  - Do math in all lectures
  - Drop notes in lectures
  - make your OWN notes
- Watch YouTube math videos
- Develop your "math brain"
  Make sure you
  - fully understand basic concepts without needing to think
  - get VERY comfortable with the terminology
- study to FULL comprehension & be able to APPLY takes PATIENCE
  Lectures are only 32 hours, you bear the responsibility of that 108 hours[2]

[2] `https://www.southampton.ac.uk/courses/modules/aice1004`

Things not-a-must-to-do but good for you

Learn LaTeX

Learn Beamer

Learn Markdown

My teaching style: no slide.

What's next: revision of high school math