# CO327 Deterministic OR Models
# Optimal transport and resource allocation

Example

Andersen Ang

Dept. Combinatorics and Optimization, U.Waterloo, Canada

emails Homepage: angms.science

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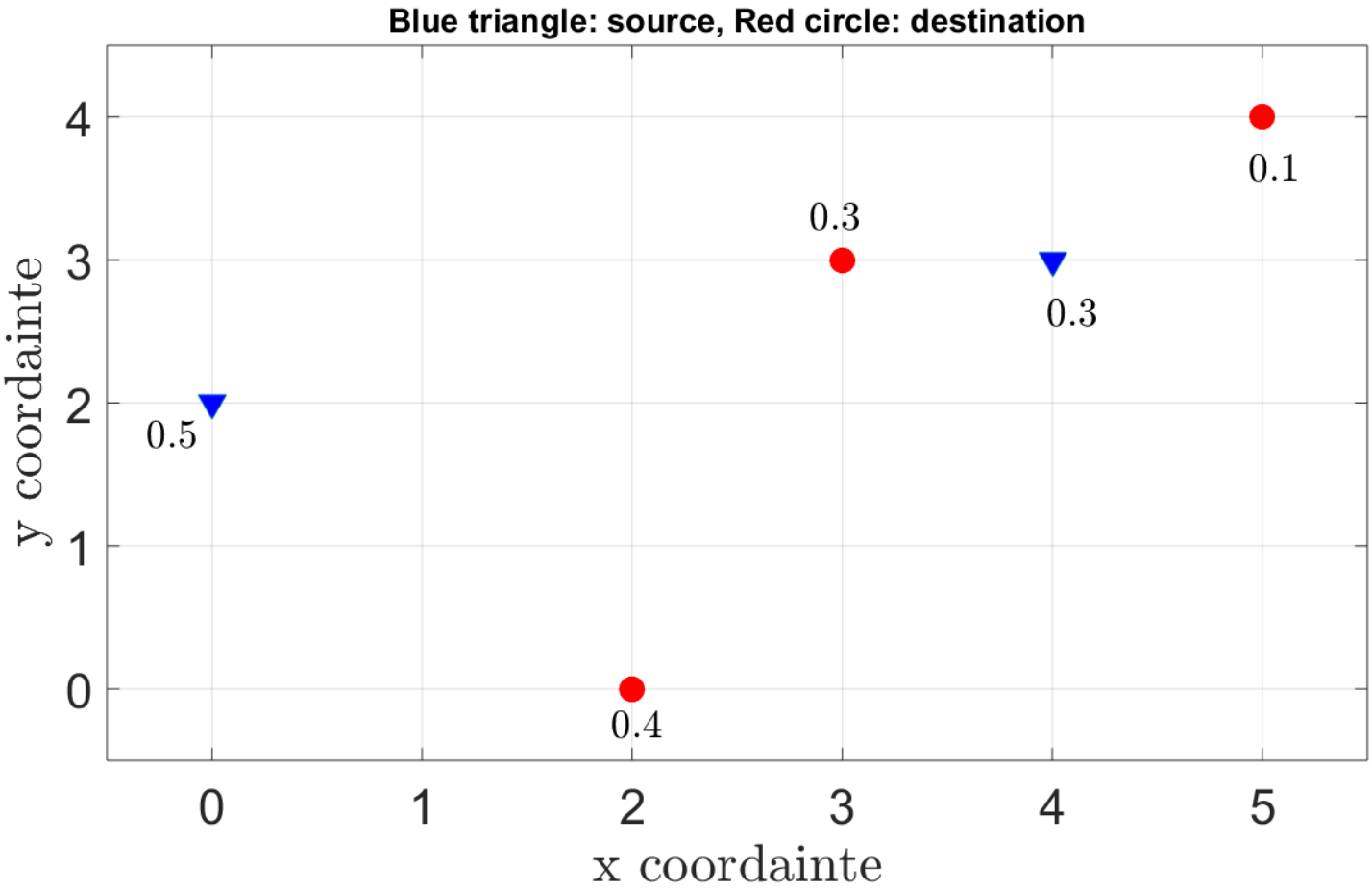
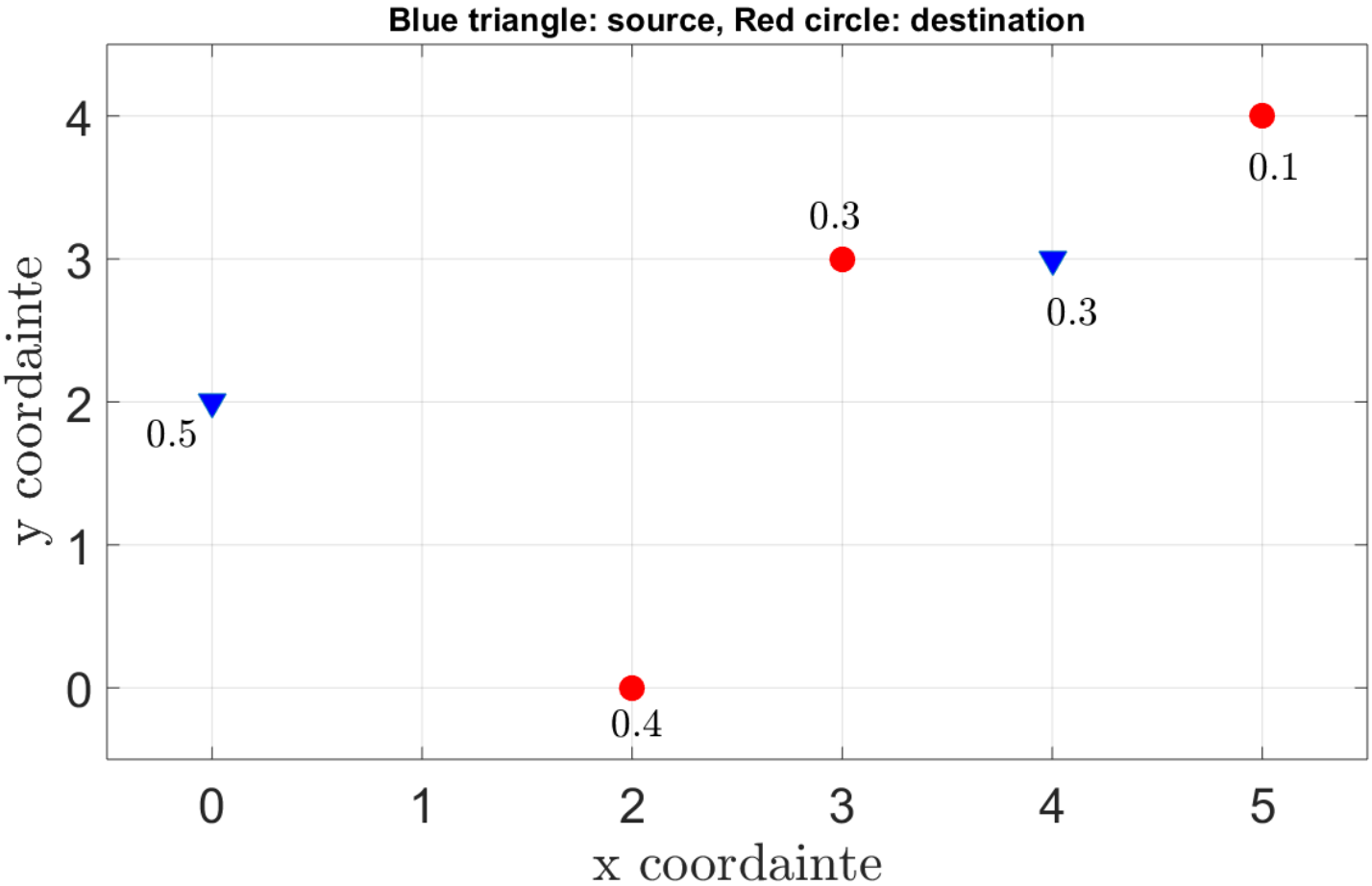
# A map



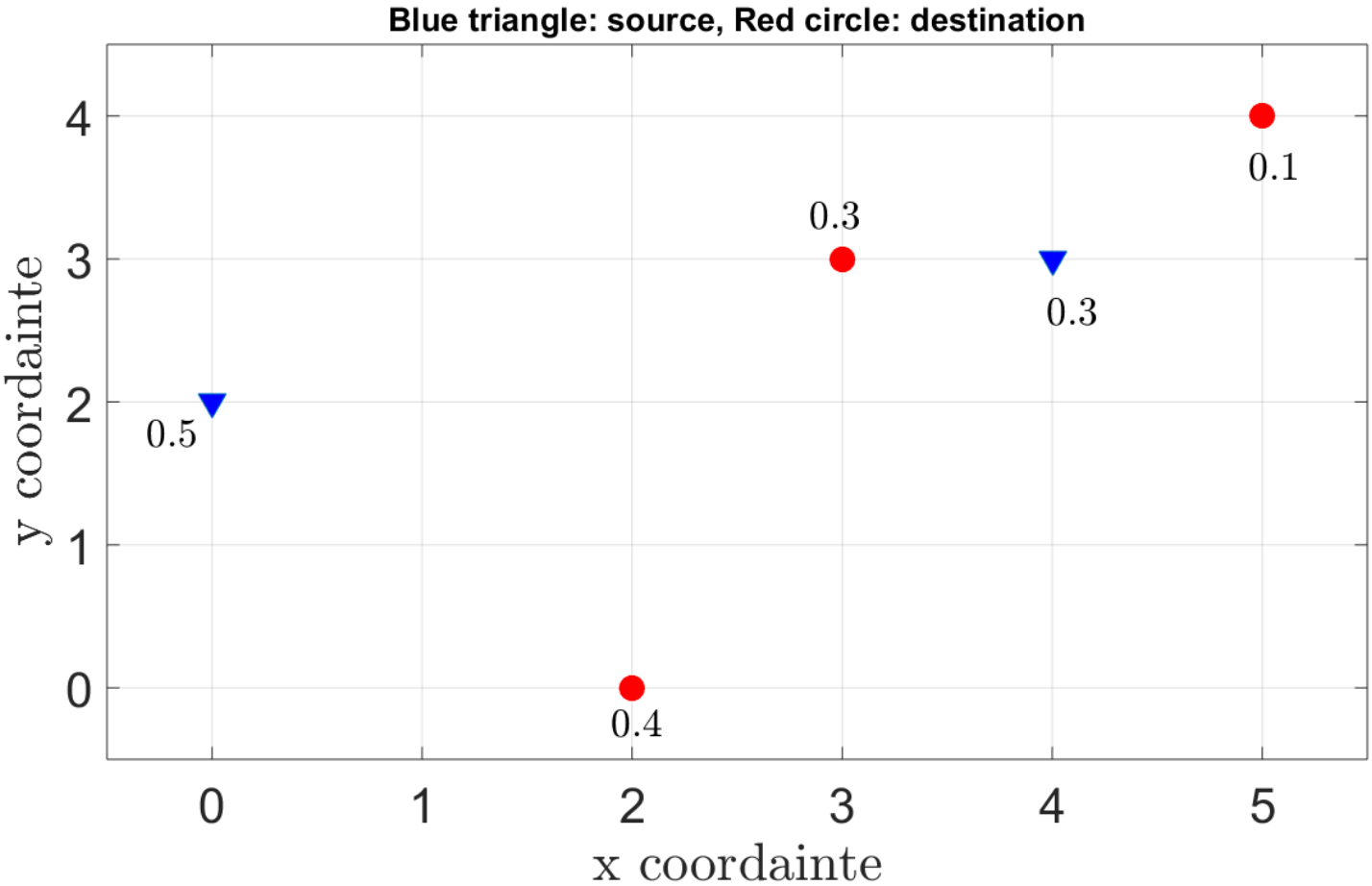

- Optimal transport problem: send products from sources to destinations.
- Cost of transportation: the distance between two points.

# Define the cost (suppose cost = distance)



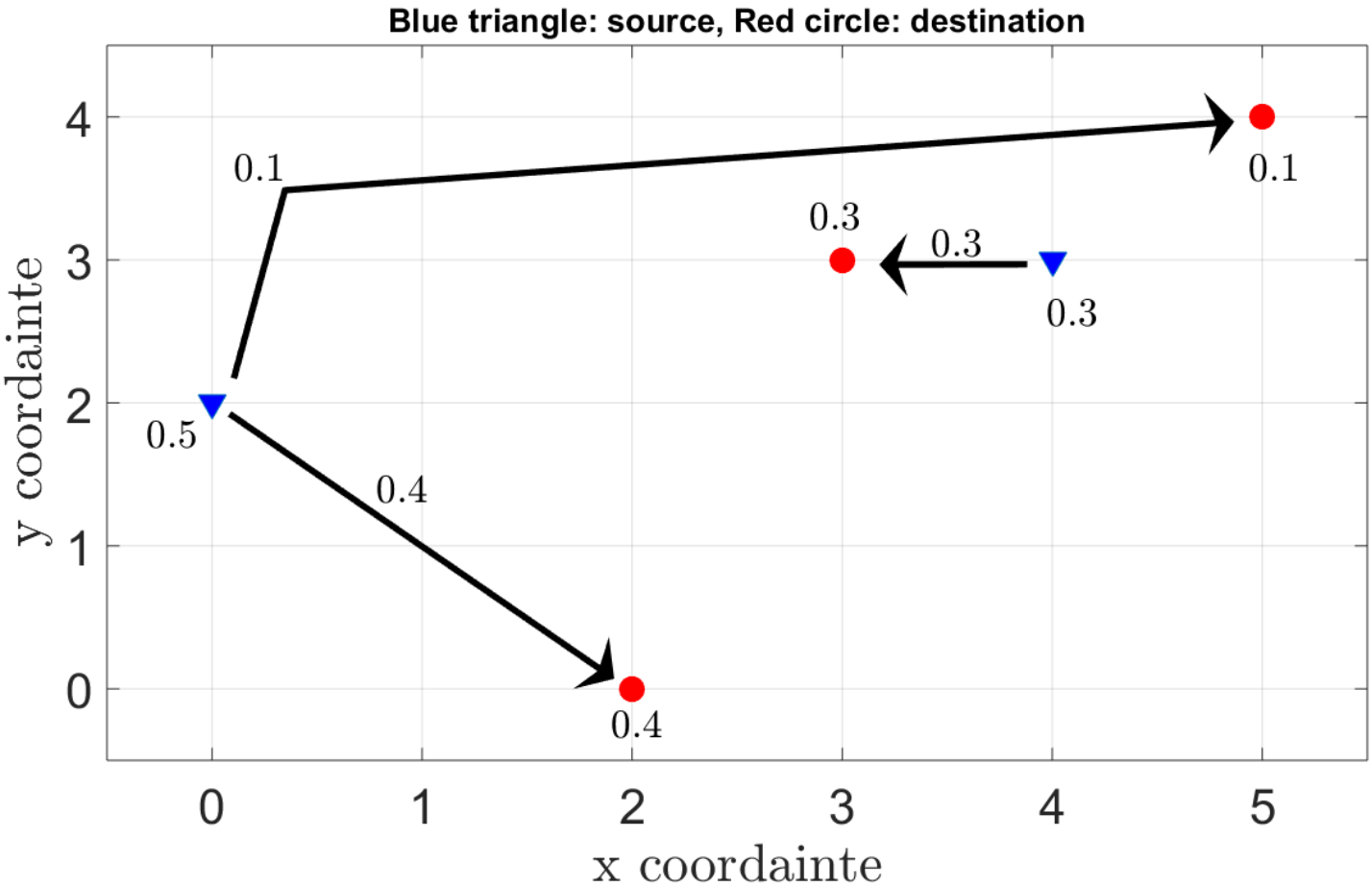

- In Euclidean distance ($l_2$ norm) between points, the cost of $(0, 2) \to (2, 0)$ is $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.

# Define the cost (suppose cost = distance)



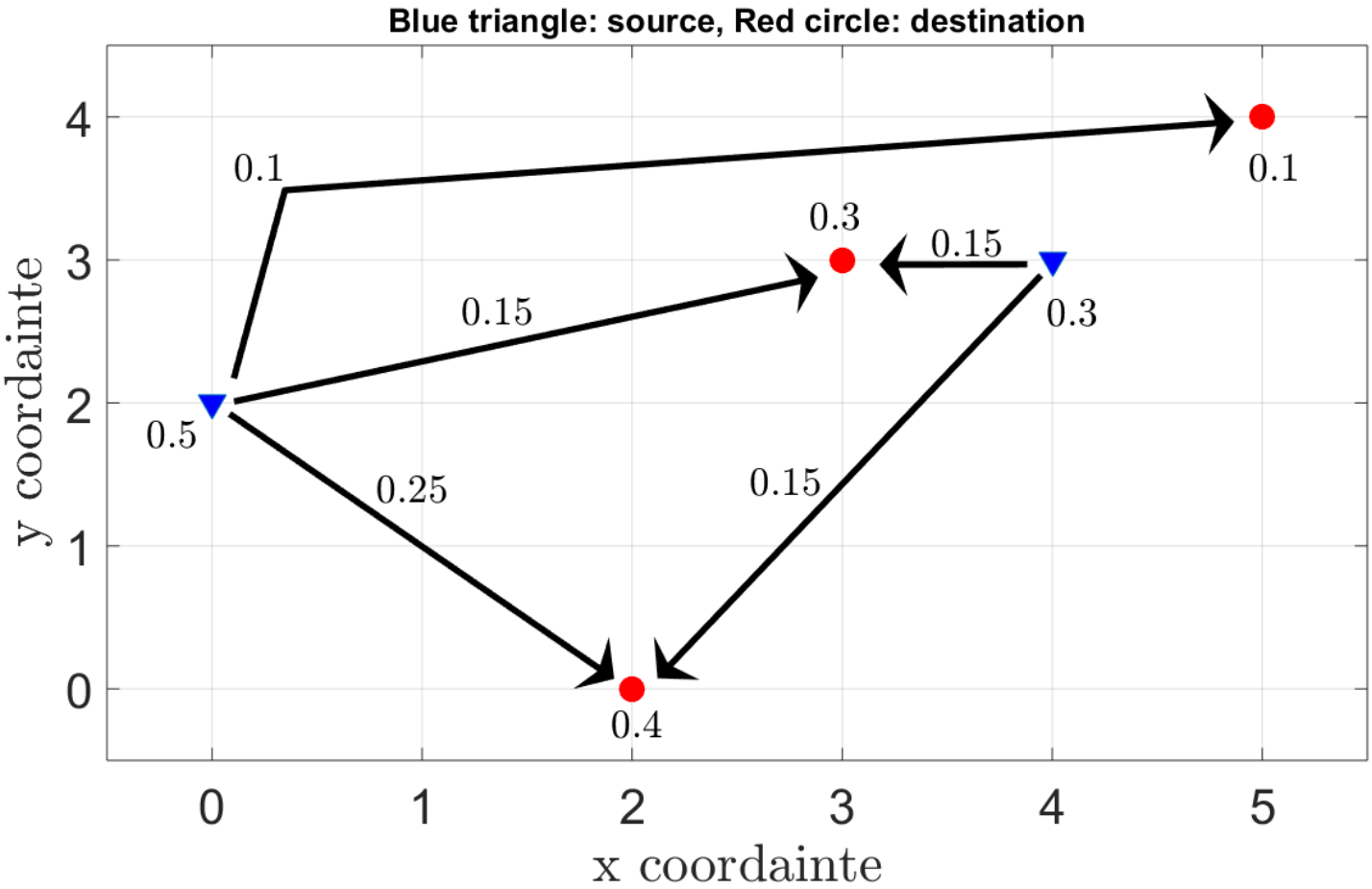

- In Manhattan distance / taxicab distance ($l_1$ norm) between points, the cost of $(0, 2) \to (2, 0)$ is $2 + 2 = 4$.

# A bad decision



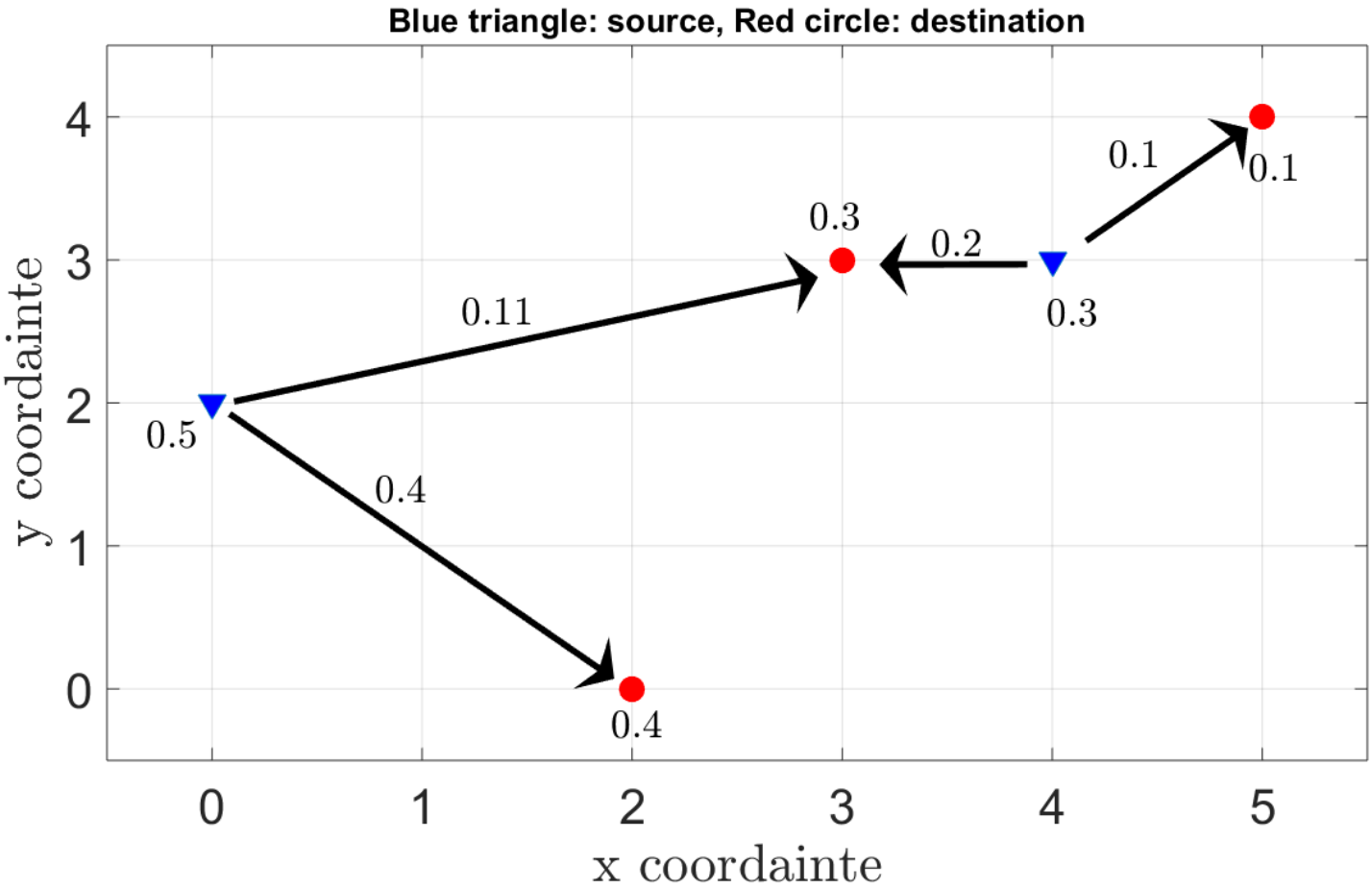

- The total cost is not minimal.

# Another bad decision



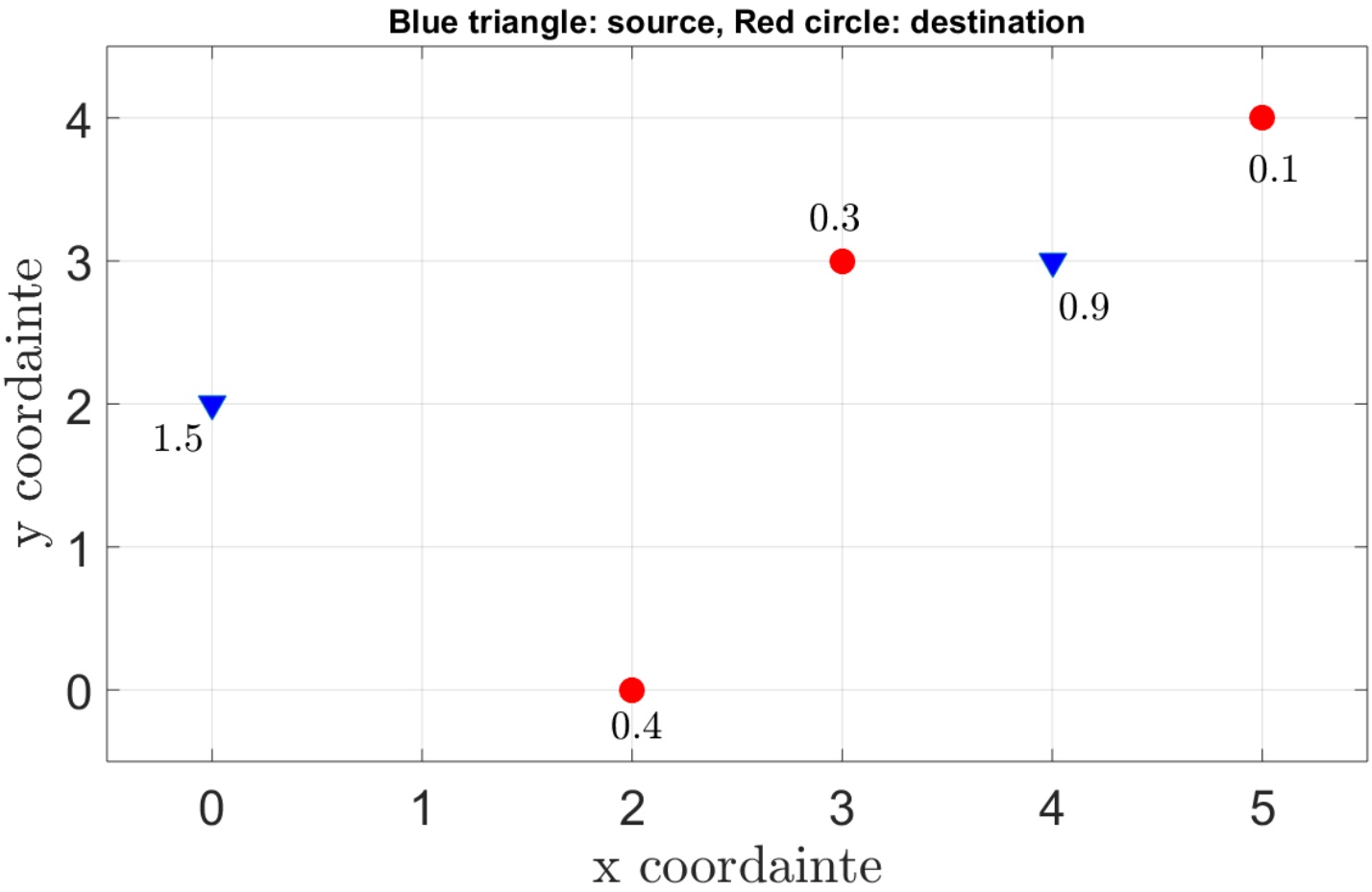

- The total cost is not minimal.

# The optimal decision




- The total cost is minimal.
- Formulate this as a LP and solve it (see assignment).

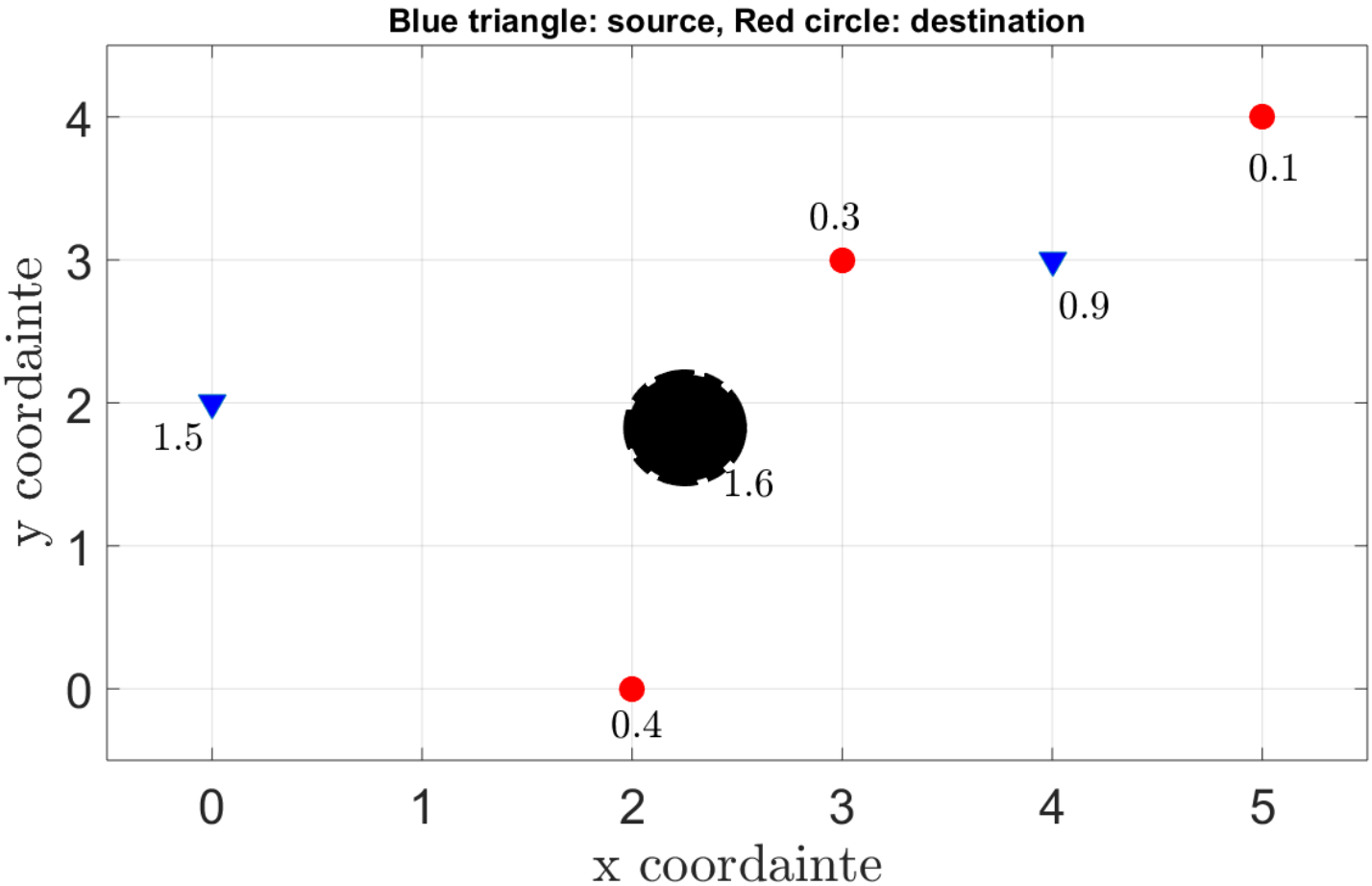
# Unbalanced optimal transport



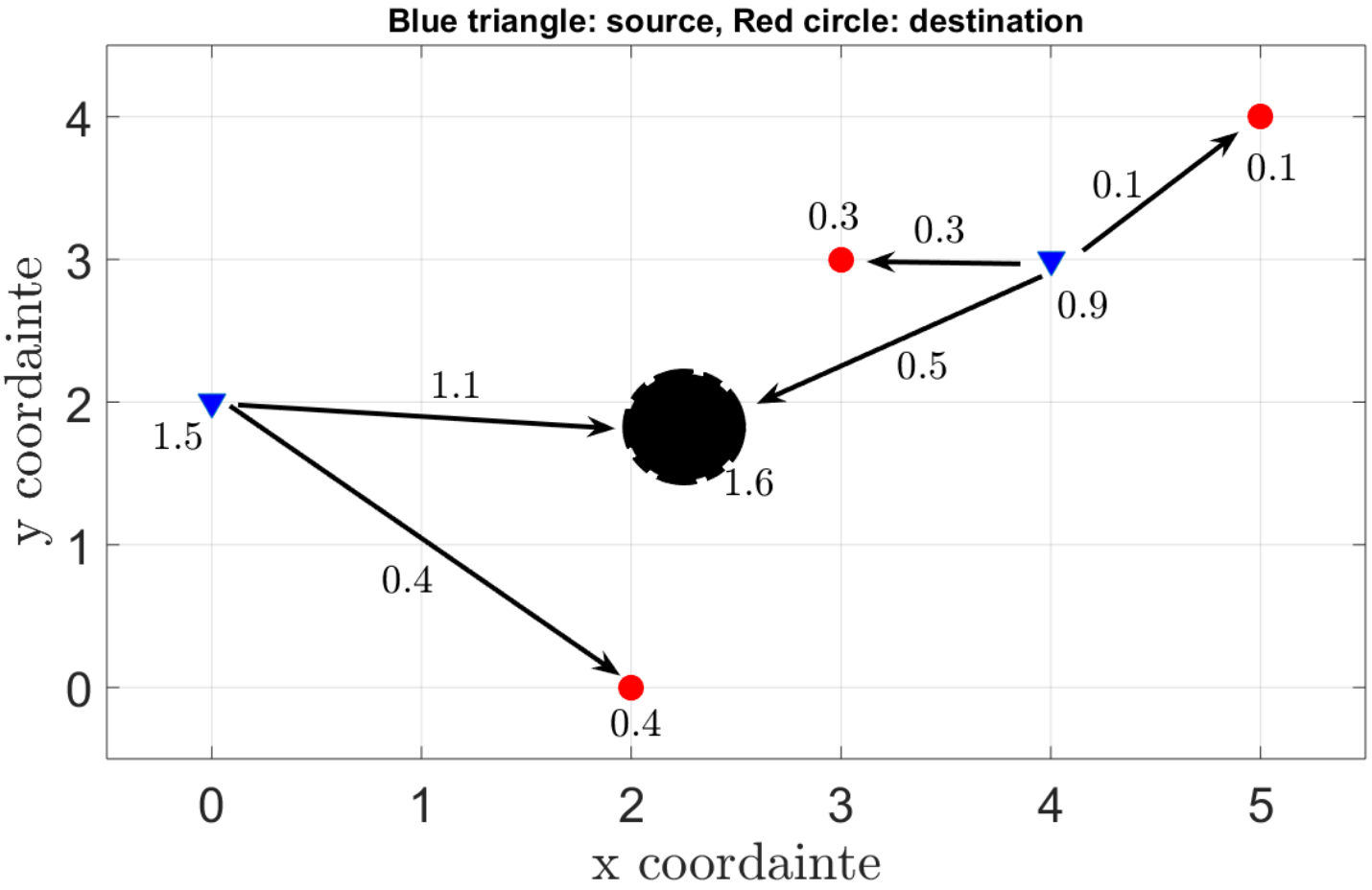

- Now more supply than demand

# Adding dummy destination

Suppose you put a black hole here...



# Is this optimal?




- Formulate this as a LP, and show this optimal (see assignment).

# What's next

- We now have a solid understanding of optimal transport.
- Next: look at the modern optimal transport in machine learning.

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