

CO327 Deterministic OR Models (2021-Spring)

MATLAB: `linprog`, `intlinprog` and `quadprog`

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How to MATLAB @ UWaterloo

- ▶ <https://uwaterloo.ca/information-systems-technology/news/expanding-campus-license-matlab>
- ▶ <https://uwaterloo.ca/coronavirus/news/matlab-programming>

To download or use MATLAB online:

1. Go to [University of Waterloo's MATLAB Portal](#).
2. Click 'Sign in to get started' under the Get MATLAB and Simulink section.
 - 2.1 If you have an existing MathWorks account (set up with your uwaterloo email), use your uwaterloo email to log in.
 - 2.2 If you do not have an existing MathWorks set up with uwaterloo email, create an account using your uwaterloo email.
3. Set up your account, link to UWaterloo MATLAB license and:
 - ▶ Download and activate MATLAB on your computer.
 - ▶ Use MATLAB Online from a Web browser without installation.

Linear programming

- ▶ The MATLAB code $\mathbf{x} = \text{linprog}(\mathbf{f}, \mathbf{A}, \mathbf{b})$ solves

$$\min_{\mathbf{x}} \mathbf{f}^T \mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}$$

- ▶ Example: solve

$$\max_{x_1, x_2} x_2 \text{ s.t. } x_1 \geq 0, x_2 \geq 0, x_1 - x_2 \geq 0, x_1 + x_2 \leq 1$$

- ▶ Solution: first turn it into canonical form

$$\mathbf{f} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

```
LP_eg1.m x LP_eg1.mlx x +
1 clear,close all,clc
2
3 f = [0; -1]
4
5 A = [-1 0;
6       0 -1;
7       -1 1;
8        1 1]
9
10 b = [0 0 0 1]';
11
12 x = linprog(f,A,b)
13
14 optimal_cost_val = f'*x
```

```
f = 2x1
    0
   -1

A = 4x2
   -1    0
    0   -1
   -1    1
    1    1

b = 4x1
    0
    0
    0
    1

Optimal solution found.
x = 2x1
    0.5000
    0.5000

optimal_cost_val = -0.5000
```

Correct (note the optimal value is negative of our original problem).

Learn more about MATLAB linprog

- ▶ Type

```
help linprog
```

for more information.

- ▶ Or visit <https://www.mathworks.com/help/optim/ug/linprog.html>

- ▶ NOTE: the interior-point (the default) algorithm of linprog uses a primal-dual method. Both the primal problem and the dual problem must be feasible for convergence. Infeasibility messages of either the primal or dual, or both, are given as appropriate.

The primal problem in standard form is

$$\min f'x \text{ such that } Ax = b, x \geq 0.$$

The dual problem is

$$\max b'y \text{ such that } A'y + s = f, s \geq 0.$$

A good habit of calling library code:

- ▶ Always call `exitflag`

```
[x,fval,exitflag,output] = linprog(f,A,b)
```

```
Optimal solution found.
```

```
x = 2×1
```

```
    0.5000
```

```
    0.5000
```

```
fval = -0.5000
```

```
exitflag = 1
```

```
output = struct with fields:
```

```
    iterations: 2
```

```
    constrviolation: 0
```

```
    message: 'Optimal solution found.'
```

```
    algorithm: 'dual-simplex'
```

```
    firstorderopt: 0
```

- ▶ Question: why MATLAB uses “dual-simplex” method here?

Linear integer program

► $\mathbf{x} = \text{intlinprog}(\mathbf{f}, \mathbf{I}, \mathbf{A}, \mathbf{b})$ solves

$$\min_{\mathbf{x}} \mathbf{f}^T \mathbf{x} \text{ s.t. } \mathbf{x}_I \in \mathbb{N}, \mathbf{Ax} \leq \mathbf{b}$$

$$\min_x 8x_1 + x_2 \text{ subject to } \begin{cases} x_2 \text{ is an integer} \\ x_1 + 2x_2 \geq -14 \\ -4x_1 - x_2 \leq -33 \\ 2x_1 + x_2 \leq 20. \end{cases}$$

```
f = [8;1];
intcon = 2;
A = [-1,-2;
     -4,-1;
     2,1];
b = [14;-33;20];
x = intlinprog(f,intcon,A,b)
```

LP: Optimal objective value is 59.000000.

Optimal solution found.

Intlinprog stopped at the root node because the objective value is within a gap tolerance of the optimal value, options.AbsoluteGapTolerance = 0 (the default value). The intcon variables are integer within tolerance, options.IntegerTolerance = 1e-05 (the default value).

```
x = 2x1
    6.5000
    7.0000
```

More on linprog

Syntax

```
x = linprog(f,A,b)
x = linprog(f,A,b,Aeq,beq)
x = linprog(f,A,b,Aeq,beq,lb,ub)
x = linprog(f,A,b,Aeq,beq,lb,ub,options)
x = linprog(problem)
[x,fval] = linprog(__)
[x,fval,exitflag,output] = linprog(__)
[x,fval,exitflag,output,lambda] = linprog(__)
```

Description

Linear programming solver

Finds the minimum of a problem specified by

$$\min_x f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

f , x , b , beq , lb , and ub are vectors, and A and Aeq are matrices.

Learn more on <https://www.mathworks.com/help/optim/ug/linprog.html>

Other programs

- ▶ Quadratic program: $\mathbf{x} = \text{quadprog}(\mathbf{H}, \mathbf{f}, \mathbf{A}, \mathbf{b})$ attempts to solve the quadratic programming problem:

$$\min_{\mathbf{x}} \frac{1}{2} \mathbf{x}^\top \mathbf{H} \mathbf{x} + \mathbf{f}^\top \mathbf{x} \text{ s.t. } \mathbf{A} \mathbf{x} \leq \mathbf{b}.$$

- ▶ What about semidefinite program and polynomial program?
You need to download external package or write your own code.
- ▶ What's now: finish assignment 4.

Study these pages

- ▶ <https://www.mathworks.com/help/optim/ug/linprog.html>
- ▶ <https://www.mathworks.com/help/optim/ug/intlinprog.html>
- ▶ <https://www.mathworks.com/help/optim/ug/quadprog.html>

What is the main difficulty of using MATLAB?

- ▶ It is not on coding but your mathematical modelling skills
- ▶ For example: the MATLAB code `x = linprog(f,A,b)` solves

$$\min_{\mathbf{x}} \mathbf{f}^T \mathbf{x} \text{ s.t. } \mathbf{Ax} \leq \mathbf{b}$$

What if your problem is

$$\min_{\mathbf{x}} \|\mathbf{f}^T \mathbf{x}\|_1 \text{ s.t. } \mathbf{Ax} \leq \mathbf{b}$$

which is not LP in its original form?

- ▶ How to solve such problem: re-formulate it as LP!

Now what: ℓ_p -norm minimization problems.

What's next section: theory time !

NOTE: the interior-point (the default) algorithm of linprog uses a primal-dual method. Both the primal problem and the dual problem must be feasible for convergence. Infeasibility messages of either the primal or dual, or both, are given as appropriate.

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