CO327 (2021Spring) Assignment 1

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- Assignment deadline: May-27 11:55 (am).
- Send your electronic copy to (`ms3ang@uwaterloo.ca`)

1 Formulation: transportation problem (10 points)

You have a company manufacturing piano. You have 2 factories located at cities F1 and F2 and 3 retail centers located at C1, C2 and C3. The monthly demand at the retail centers are 8, 5 and 2 respectively while the monthly supply at the factories are 6 and 9 respectively. Note that the total supply equals the total demand. You are also given the cost of transportation of 1 piano between any factory and any retail center.

	C1	C2	C3
F1	5	5	3
F2	6	4	1

Table 1: Cost of transportation.

Your goal is to determine the quantity to be transported from each factory to each retail center so as to meet the demand at minimum total shipping cost. Formulate this problem as a linear program/integer program. State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program.

2 Formulation: knapsack problem (6 points)

You plan to go on a camping trip. There are 5 items you wish to take with you, but together they exceed the 60 weight limit you can carry. You assigned a value to each item:

Item	1	2	3	4	5
Weight	52	23	35	15	7
Value	100	60	70	15	15

Table 2: Weight and value of each item.

Your goal is to determine which item to take to maximize the total value without exceeding the weight restriction. Formulate this problem as a linear program/integer program. State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program.

3 Formulation: employee worksheet (8 points)

You are the boss of a 24-hour supermarket. The supermarket has the following minimal requirements for cashiers:

Period	1	2	3	4	5	6
Time of the day (24-h)	3-7	7-11	11-15	15-19	19-23	23-3
Wages	110	100	100	107	107	110
Minimum no. needed	2	10	4	12	4	2

Table 3: Work period, wages and minimum number of cashiers.

Period 1 follows immediately after period 6. A cashier works 8 consecutive hours, starting at the beginning of one of the six periods. Different periods have different wages. Your goal is to determine a daily employee worksheet which satisfies the requirements with the least wages. Formulate this problem as a linear program/integer program. State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program.

4 Formulation: traveling salesman problem (7 points)

You are a salesman. You need to travel all 5 cities (C1,C2, ..., C5) exactly once to sale your product. The distance between the cities are shown below.

	C1	C2	C3	C4	C5
C1	M	28	57	20	45
C2	28	M	47	46	53
C3	57	47	M	76	85
C4	20	46	M	M	40
C5	45	73	M	40	M

Table 4: Distances between the cities

According to the table, we can let c_{ij} be the distance from city i to city j ($i, j \in \{1, 2, 3, 4, 5\}$). Your goal is to determine the order of visiting that minimize the total distance traveled.

About M Physically the distance between C1 and C1 is zero so naturally we should put $c_{11} = 0$. However this will lead to traveling from C1 to C1 (so we will trap inside C1 forever). Therefore, for proper modeling, M is a big number (e.g., 999).

Traveling Salesman Problem (TSP) is a famous integer programming problem and can be formulated as follows. Let $x_{ij} = 1$ denotes the decision of traveling from city i to city j, and let $x_{ij} = 0$ denotes the decision of NOT traveling from city i to city j. That is, x_{ij} is a nonnegative integer variable in $\{0, 1\}$.

4.1 Using x_{ij} and the table, write down the objective function of this problem (1 point). Is TSP a minimization or maximization problem? (2 points).

4.2 A constraint of TSP is that you need to enter all cities exactly once. Write down this constraint mathematically. (1 point).

4.3 A constraint of TSP is that you need to leave all cities exactly once. Write down this constraint mathematically. (1 point).

4.4 Are the two constraints mentioned above completely describe the constraint set of TSP? Is there any missing constraint? If yes, comment about it (2 point).

5 Transform to canonical form and standard form (22 points)

5.1 Transform the LP into canonical form, shows the working steps (8 points).

$$\begin{aligned} \min_{x_1,x_2,x_3} \quad & 0 \\ \text{s.t.} \quad & 1x_1 + 3x_2 = 5 \\ & 2x_1 + 4x_3 \geq 6 \\ & 2 \leq x_2 \leq 4 \end{aligned}$$

Also write down the vector $\mathbf{c}, \mathbf{b}$ and matrix $\mathbf{A}$.

5.2 Transform the LP into standard form (7 points)

$$\begin{aligned} \min_{x_1,x_2} \quad & 5x_1 + 2x_2 \\ \text{s.t.} \quad & 6x_1 + x_2 \geq 6 \\ & 4x_1 + 3x_2 \leq 12 \end{aligned}$$

5.3 Transform the LP of 5.1 to standard form (7 points).

6 Solving LP graphically (11 points)

6.1 Draw the constraint set and solve the LP (5 points).

$$\min_{\mathbf{x}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}^\top \mathbf{x} \, \text{s.t.} \, \mathbf{x} \in \mathcal{C} = \begin{cases} x_1 + x_2 \geq 1 \\ x_1 + x_2 \leq 2 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

6.2 Make it as simple as you can, write down a LP that has no solution, and draw the constraint set (2 points)

6.3 Make it as simple as you can, write down a LP that the constraint set is unbounded, and draw the constraint set (2 points)

6.4 Make it as simple as you can, write down a LP that the optimal point is non-unique, and draw the constraint set (2 points)

END of assignment 1.