CO327 (2021Spring) Assignment 2

Lecturer: Andersen Ang

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Assignment deadline: **June-1 11:25 (am)**. Drop your electronic copy (with your name and your student number) to the dropbox folders in the course page (CO327 - Spring 2021) in WATERLOO LEARN (`learn.uwaterloo.ca`).

1 Formulation: Fuel purchase policy making (10 points)

Your airline company can buy jet fuel from any one of three vendors. The airline's need for the upcoming month at each of the three airports it serves are 100000 unit at airport 1, 180000 unit at airport 2, and 350000 unit at airport 3. Each vendor can supply fuel to each airport at a price ($ per unit) given by the following table

	Airport 1	Airport 2	Airport 3
Vendor 1	92	89	90
Vendor 2	91	91	95
Vendor 3	78	90	82

Each vendor, is limited in the total number of unit of fuels it can provide during any one month. These capacities are 32000 unit for vendor 1, 270000 units for vendor 2, and 190000 units for vendor 3.

Determine a purchasing policy that will supply the airline's requirement at each airport at minimum total cost, write down the optimization problem (linear program/integer program). State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program

2 Formulation: Resource allocation (18 + 2 points)

2.1 Voting visit (5 + 2 points)

You are the next US presidential candidate. You must decide which states to visit in the 10 days before the election. Your goal is to increase the number of votes by the largest possible amount. Your election team provide you the following data
Formulate this problem. State clearly your decision variable(s), objective function and constraint(s).
Bonus (2 points): Which states should be visited? How many votes will be generated by these visits?

2.2 Voting visit and ads (13 points)

Now you are down to the last 5 days of the campaign. You have $300000 left and three key states appear likely to swing the election one way or other. Each state can be visited, or a TV ad series can be purchased. Your election team provide you the following data

Now assume

State	Vote ($\times 10^3$) increase by visit	Days required for visit
1	10	4
2	20	3
3	40	3
4	90	4
5	30	3
6	10	1

State	Action	Vote ($\times 10^3$) increase by visit	Days required for visit	Cost ($\times 10^3$)
1	Visit	100	4	200
1	Ads	50	0	100
2	Visit	80	4	150
2	Ads	40	0	90
3	Visit	20	1	45
3	Ads	15	0	30

- case a: visit and ads on the same state are not mutually exclusive (6 points)
- case b: visit and ads on the same state are mutually exclusive (7 points)

Formulate these problems. State clearly your decision variable(s), objective function and constraint(s).
* You do not need to solve the program.

3 Formulation: Tool assignment problem (10+4 points)

You are a miner and you mine two ores called red stone and blue stone. To achieve this, you make two tools: a pickaxe and a drill, out of two material: iron or diamond.

- Because of the weights of the tools to carry, you can only bring one at most pickaxe and one drill for the mining.
- Because of the scarcity of the raw material, you can only make at most one iron tool and at most one diamond tool.
- Because of the stamina constraint, you can only use a tool to mine one kind of ore. That is, if you choose to use a drill to mine red stone, you cannot use the same drill to mine blue stone.

The table below shows how many ore the tool can mine in the same amount of time.

	Made of iron			Made of diamond	
	Pickaxe	Drill		Pickaxe	Drill
Red stone	5	10	Red stone	20	40
Blue stone	10	12	Blue stone	30	50

You task is to assign which tool to make and to use which tool for mining which specific ore, such that the total amount of ore is maximized. Note that, both ores have to be mined. Formulate this as a integer programming problem. State clearly your decision variable(s), objective function and constraint(s).

Bonus (4 points) Solve this problem, state the best option (which type of tool to mine which ore, and what is the total number of ores can be mined), repeat for the **second best** option.

4 Formulation: Set cover problem of Tower defense (29 points)

(This question requires the knowledge of tower defense discussed in lecture 2)

Consider the map in Fig.1 (follows the labeling of the grid). You are allowed to place turret on the labeled blocks, while the red blocks are the path for the enemy. You goal is to determine how to place the least number of turrets, such that each red block is targeted by at least 1 turret.

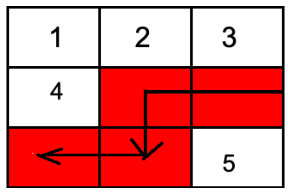

Figure 1: A map.

4.1 Turret can hit all the 4 adjacent blocks (7 points)

Here is the specification of the turret: after placing a turret at a block, the turret can hit all the 4 adjacent blocks – N,E,S,W (North N, East, South, West) simultaneously at once. Formulate the problem as a BIP. Solve this BIP by brute-force and list all the optimal solution(s), if there is no solution, explain why.

4.2 Rocket launcher (9 points)

Now, beside turrets, you can place rocket launcher in the map. Turrets and rocket launcher are collectively called as "towers". The specifications of the game changed as follows:

- For all the turrets, after placing at a block, the turret can hit all the 4 adjacent blocks (N,E,S,W) simultaneously at once.
- For all the rocket launchers, after placing at a block, the rocket launcher can hit all the 8 adjacent blocks (N,E,S,W,NE,NW,SE,SW) simultaneously at once.
- You can not place a turret and a rocket launcher at the same block.
- Building a turret costs 1 gold, while building a rocket launcher costs 2 gold.

Your goal is to determine how to place the of turrets and rocket launcher with the minimum cost such that each red block is targeted by at least 2 towers. Formulate the problem as a BIP. Solve this BIP by brute-force and list all the optimal solution(s), if there is no solution, explain why.

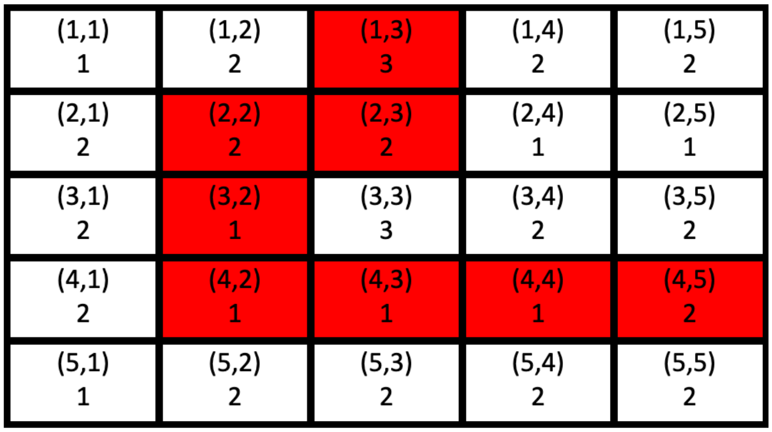

Figure 2: Another map.

4.3 Another map (13 points)

Consider the map in Fig.2.

- The map is 5-by-5 grid. Each box is labeled with (i, j) for the coordinate.
- The enemy will walk pass the red blocks, your task is to place turrets on the white blocks to defend the enemy. You cannot place turrets on the red blocks.
- The number in white block is the amount of gold you need to pay if you build one turret on that block. The number in the red block is the number of turrets needed to target such block.
- You can place multiple number of turrets on the same block, and you can place at most 10 turrets in each block.
- After placing a turret at a block, all turrets can hit all the 4 adjacent (N,E,S,W) blocks.

You goal is to determine how to place the turrets with the lowest cost, such that the red path is safe. Write down the optimization problem (linear program/integer program). State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program

5 Computation (8 points)

Transform the following IP

$$\max\ -x_1 + 2x_2 + 3x_3 \text{ s.t. } 4x_1 - 5x_2 + 6x_3 \geq -7, x_i \text{ is integer}, x_i \geq 0$$

into a integer Knapsack problem in the form

$$\max\ \mathbf{c}^\top \mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \text{ is integer vector}$$

where $\mathbf{A}, \mathbf{b}, \mathbf{c}$ are elementwise nonnegative. Write down the $\mathbf{A}, \mathbf{b}$ and $\mathbf{c}$. Solve the Knapsack problem: state the optimal decision variable and the optimal cost value.

END of assignment.