Solution to CO327 (2022Spring) Assignment 2

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1 Formulation: transportation problem (10 points)

You have a company manufacturing piano. You have 2 factories located at cities F1 and F2 and 3 retail centers located at C1, C2 and C3. The monthly demand at the retail centers are 8, 5 and 2 respectively while the monthly supply at the factories are 6 and 9 respectively. Note that the total supply equals the total demand. You are also given the cost of transportation of 1 piano between any factory and any retail center.

	C1	C2	C3
F1	5	5	3
F2	6	4	1

Table 1: Cost of transportation.

Your goal is to determine the quantity to be transported from each factory to each retail center so as to meet the demand at minimum total shipping cost. Formulate this problem as a linear program/integer program **in standard form**. State clearly your decision variable(s), objective function and constraint(s).

*You do not need to solve the program.

1.1 Solution

2 factories and 3 retail centers: 6 decision variable encoding how much each factory ship to each city. Let x_{ij} be shipping from ith city to jth retail center, the decision variables are

$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}. \qquad \text{(1 point, 1/6 each)}$$

The objective function is then the total shipping cost

$$5x_{11} + 5x_{12} + 3x_{13} + 6x_{21} + 4x_{22} + 1x_{23} \qquad \text{(1 point, either 1 or 0)}$$

Explicit constraint 1: meet the demand of retail centers

$$\begin{aligned} x_{11} + x_{21} &= 8 \quad \text{meet the demand of retail centers at C1} \\ x_{12} + x_{22} &= 5 \quad \text{meet the demand of retail centers at C2} \\ x_{13} + x_{23} &= 2 \quad \text{meet the demand of retail centers at C3} \end{aligned} \qquad \text{(3 points, 1 point each)}$$

Explicit constraint 2: meet the supply of factories

$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 6 \quad \text{meet the supply of factory at F1} \\ x_{21} + x_{22} + x_{23} &= 9 \quad \text{meet the supply of factory at F2} \end{aligned} \qquad \text{(2 points, 1 point each)}$$

Implicit constraint 1: you ship nonnegative amount of piano

$$x_{11} \geq 0, x_{12} \geq 0, x_{13} \geq 0, x_{21} \geq 0, x_{22} \geq 0, x_{23} \geq 0. \quad \text{(1 point, 1/6 each)}$$

Implicit constraint 2: you don't make 0.5 piano

$$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \text{ are integer.} \quad \text{(1 point, 1/6 each)}$$

So the LP is

$$\begin{aligned} \min_{x_{11},\ldots,x_{23}} \quad & 5x_{11} + 5x_{12} + 3x_{13} + 6x_{21} + 4x_{22} + 1x_{23} \\ \text{s.t.} \quad & x_{11} + x_{21} = 8 \\ & x_{12} + x_{22} = 5 \\ & x_{13} + x_{23} = 2 \\ & x_{11} + x_{12} + x_{13} = 6 \\ & x_{21} + x_{22} + x_{23} = 9 \\ & x_{11} \geq 0, x_{12} \geq 0, x_{13} \geq 0, x_{21} \geq 0, x_{22} \geq 0, x_{23} \geq 0 \\ & x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \text{ are integer.} \end{aligned}$$

(1 point for the min)

(10 points if the student directly write down the LP and it is all correct. Otherwise, deduct the points according to the missing portion.)

2 Cargo plane loading (17 points)

You own a logistic company and you have a cargo plane with 1000 meter^3 of space available. Five suppliers approach you and want you to deliver their cargo to their desired destination. After negotiating with these suppliers, you can choose from a set of 5 cargo to load the plane. The table on the right lists the volume and profit from shipping each of the cargo.

Cargo type	Volume	Profit
1	410	200
2	600	60
3	220	20
4	450	40
5	330	30

Now, suppose there are at most 1 cargo available from each supplier, and:

2.1 we are allowed to take any portion of each cargo up to the total volume, with the profit adjusted accordingly (that is, if you take half of the cargo, it yields half the profit). Formulate this problem as as a linear program. State clearly your decision variable(s), objective function and constraint(s).
*You do not need to solve the program.

2.2 we must take all of the cargo or none of it. Model such cargo plane loading problem as an linear integer program. Solve this problem by brute force: enumerating all the possible solutions and find the best one.

Sol 2.1 Let

- a_j be the volume of the jth cargo
- c_j the profit
- $b = 1000$ the total volume available in the cargo plane.

Introduce the decision variable x_j to represent the amount of the jth cargo that we decide to take.
The LP is

$$\max \quad \mathbf{c}^\top \mathbf{x} \text{ s.t. } \mathbf{a}^\top \mathbf{x} \le b, \quad \mathbf{x} \ge \mathbf{0} \quad \text{and } \mathbf{x} \in \mathcal{C}$$

where

$$\mathbf{c} = \begin{bmatrix} 200 \\ 60 \\ 20 \\ 40 \\ 30 \end{bmatrix} \qquad \mathbf{a} = \begin{bmatrix} 410 \\ 600 \\ 220 \\ 450 \\ 330 \end{bmatrix} \qquad b = 1000$$

and $\mathcal{C} = \{\mathbf{x} \mid \mathbf{0} \le \mathbf{x} \le \mathbf{1}\}$.
Marking: 7 points in total

- "max" in the cost function 1 point
- correct $\mathbf{c}$ 1 point
- correct $\mathbf{a}$ 1 point
- correct b 1 point
- constraint $\mathbf{a}^\top \mathbf{x} \le b$ 1 point
- constraint $\mathbf{x} \ge \mathbf{0}$ 1 point
- constraint $\mathbf{x} \in \mathcal{C}$ with correct $\mathcal{C}$ 1 point

(Also accept LP expressed in standard form or canonical form.)

2.2 Same LP with $\mathcal{C} = \{0,1\}^5$. The optimal solution: $\mathbf{x}^* = [1,0,1,0,1]^\top$, and optimal cost $= 250$.
Marking:

- correct optimization problem 2 points
- the 2^5 brute force possibilities 6 points
- correct $\mathbf{x}^*$ 1 point
- correct optimal cost 1 point

cargo choice					profit
1	2	3	4	5	
0	0	0	0	0	0
0	0	0	0	1	30
0	0	0	1	0	40
0	0	0	1	1	70
0	0	1	0	0	20
0	0	1	0	1	50
0	0	1	1	0	60
0	0	1	1	1	90
0	1	0	0	0	60
0	1	0	0	1	90
0	1	0	1	0	0
0	1	0	1	1	0
0	1	1	0	0	80
0	1	1	0	1	0
0	1	1	1	0	0
0	1	1	1	1	0
1	0	0	0	0	200
1	0	0	0	1	230
1	0	0	1	0	240
1	0	0	1	1	0
1	0	1	0	0	220
1	0	1	0	1	250
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	0

END of assignment 2