CO327 (2022Spring) Assignment 3

Lecturer: Andersen Ang

May 9, 2022

The goal of this assignment is to familiarize yourself with calling library/solver in MATLAB.

The solvers we will use

- Linear programming: the code `x = linprog(f,A,b)` will solve

$$\min_{\mathbf{x}} \ \mathbf{f}^\top \mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b},$$

  for $\mathbf{f} \in \mathbb{R}^n$ (n-by-1 column vector), $\mathbf{x} \in \mathbb{R}^n$ (n-by-1 column vector), $\mathbf{A} \in \mathbb{R}^{m \times n}$ (m-by-n matrix) and $\mathbf{b} \in \mathbb{R}^m$ (m-by-1 column vector).

- Linear Integer programming: the code `x = intlinprog(f,I,A,b)` will solve

$$\min_{\mathbf{x}} \ \mathbf{f}^\top \mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \ \mathbf{x}_I \in \mathbb{N}$$

  for $\mathbf{f} \in \mathbb{R}^n$, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$ and I is a subset of $\{1, 2, \dots, n\}$ labelling which element of $\mathbf{x}$ has to be an integer.

Remark

- The problem in the original form may not have the same expression as the one used in these solvers, your first task is to convert your problem into the form used by the solver. Be careful with the max and min, and the direction of the inequality signs.

- You can also study these codes and make use of their advanced structure. For example, the code `x = linprog(f,A,b,Aeq,beq,l,b)` will solve

$$\min_{\mathbf{x}} \ \mathbf{f}^\top \mathbf{x} \text{ s.t. } \mathbf{A}\mathbf{x} \leq \mathbf{b}, \ \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq}, \ \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}.$$

  where $\mathbf{A}_{eq}, \mathbf{b}_{eq}$ refer to the constants for the equality constraints and $\mathbf{l}, \mathbf{u}$ are lower bound and upper bounds. For empty input, the square bracket [ ] can be used.
  For example, the code `x = linprog(f,[],[],Aeq,beq,[],u)` will solve

$$\min_{\mathbf{x}} \ \mathbf{f}^\top \mathbf{x} \text{ s.t. } \ \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq}, \ \mathbf{x} \leq \mathbf{u}.$$

- Make sure the variables in your code have the correct dimension. Be careful of transpose (apostrophe in MATLAB), do not mix up column vector and row vector.

Submission of assignment You submit two things: the MATLAB m-files and a pdf.

- The MATLAB m-files contain the code that solve the problem. The m-file should contains all the variable (those $\mathbf{f}, \mathbf{A}, \mathbf{b}$, etc discussed above). The m-file has to be "run-able" and has no bug. You get points when the m-file can be run in MATLAB and show the correct solution.

- The pdf contains the answer of the questions, plus explanation of the your code (if you think extra explanation is needed).

Zip your mfiles and pdf into one single .7z (or .zip) file with the file name:

`your student ID+‘‘underscore’’+ a3`

and submit to the dropbox in WATERLOO LEARN, with the **deadline**: 2022-June-5 23:55.

1 Simple problems (9 points)

Find the optimal $\mathbf{x}$. If the problem has no solution, explain why.

(1)
$$\begin{aligned} \max_{x_1,x_2} \quad & 7x_1 + 5x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \le 100 \\ & 4x_1 + 3x_2 \le 240 \\ & x_1 \ge 0, x_2 \ge 0 \end{aligned}$$

(2)
$$\begin{aligned} \min_{x_1,x_2,x_3} \quad & -5x_1 - 4x_2 - 6x_3 \\ \text{s.t.} \quad & x_1 - x_2 + x_3 \le 20 \\ & -3x_1 - 2x_2 - 4x_3 \ge -42 \\ & 3x_1 + 2x_2 \le 30 \\ & x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{aligned}$$

(3)
$$\begin{aligned} \max_{x_1,x_2} \quad & 150x_1 + 175x_2 \\ \text{s.t.} \quad & 7x_1 + 11x_2 \le 77 \\ & 10x_1 + 8x_2 \le 80 \\ & 1 \le x_1 \le 9 \\ & 3 \le x_2 \le 6 \end{aligned}$$

2 LP relaxation of IP (10 points)

1. Draw the feasible region of the following constraints: $x_1 + x_2 \le 6$, $5x_1 + 9x_2 \le 45$, $x_1 \ge 0, x_2 \ge 0$.
2. Find $\mathbf{x}_{LP} = [x_1, x_2]^\top$ that maximizes $5x_1+8x_2$ subject to $x_1+x_2 \le 6$, $5x_1+9x_2 \le 45$, $x_1 \ge 0, x_2 \ge 0$. If the problem has no solution, explain why.
3. Find $\mathbf{x}_{IP} = [x_1, x_2]^\top$ that maximizes $5x_1+8x_2$ subject to $x_1+x_2 \le 6$, $5x_1+9x_2 \le 45$, $x_1 \ge 0, x_2 \ge 0$ and x_1, x_2 are integers. If the problem has no solution, explain why.
4. (On LP relaxation) Round the solution $\mathbf{x}$ for the LP problem to:
   - The closest integer $\mathbf{x}_{LP}^{Round}$
   - The closest integer that is feasible for the constraint $\mathbf{x}_{LP}^{Round,feasible}$

   Let $f(\mathbf{x})$ be the objective function values at $\mathbf{x}$. Compare the following values: $f(\mathbf{x}_{LP})$, $f(\mathbf{x}_{IP})$, $f(\mathbf{x}_{LP}^{Round})$ and $f(\mathbf{x}_{LP}^{Round,feasible})$. Explain why these objective function values are different. What can you tell about the effectiveness of LP relaxation on solving this IP?

3 BIP (6 points)

1. Find $\mathbf{x} = [x_1, x_2, x_3, x_4]^\top$ that maximizes $8x_1+11x_2-6x_3+4x_4$ subject to $5x_1+7x_2-4x_3+3x_4 \le 14$ and all $x_i \in \{0, 1\}$. If the problem has no solution, explain why.
2. Convert the previous problem into a Knapsack problem and solve the new BIP. Does the new problem share the same solution to the previous one? If no, explain why.

END.