

CO327 (2022Spring) Assignment 5

Lecturer: Andersen Ang

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- Assignment deadline: June-23 23:55.
- Submit your electronic copy (in a single PDF) to the dropbox in Waterloo LEARN.

1 Making dogs happy / Welfare policy (11 points)

You have 3 dogs (D1, D2, D3) and you give toys to them. Since you are a busy person, you have decided to give the same number of toys to each dog. Let x be the number of toys each dog receives. As you are very familiar with the dogs, you know the happiness level of each dog as a function of the number of toy it receives:

$$D1 : 1 + 2x, \quad D2 : 2 + x, \quad D3 : 5 - x.$$

You have a budget limit that you can give each dog at most 5 toys. To be fair to the dogs, you want to maximize the minimum happiness level of all the dogs.

1 Let $f(x)$ be the minimum happiness level of all 3 dogs when each dog receives x toys. Write down the expression of $f(x)$. Draw $f(x)$ for $x \in \{0, 1, 2, 3, 4, 5\}$.

2 Translate this problem to an optimization problem. Solve for the optimal x and list the happiness level of the dogs.

3 Now, instead of maximizing the minimum happiness level of all 3 dogs, you decided to maximize the **sum** of the happiness factors of all 3 dogs. Write down the optimization problem. What is the optimal x ? What are the happiness level of the dogs?

4 Consider give toys to dogs as an analogy to deciding a welfare policy. Now “toys” are welfare support and “dogs” are people. Comparing the approaches of (2) and (3) and their solution, which approach you prefer? Why?

2 Scheduling problem (13 points)

You have a train company and you want to determine the arrival time of 3 trains. Let t_i be the arrive time of the i th train, the time t_i are arranged in ascending order $0 \leq t_1 \leq t_2 \leq t_3 \leq +\infty$. However, for safety reason, you want to maximize the smallest time gap between *any two consecutive trains*, subject to time range constraints $l_i \leq t_i \leq u_i$, $i = 1, 2, 3$, where

i	1	2	3
l_i	0	1	2
u_i	2	3	4

Write down the optimization problem in *canonical form*. Solve this optimization problem. If the problem has no solution, explain why.

MATLAB hints When you constructing the constants $\mathbf{A}, \mathbf{b}, \mathbf{c}$, the code

- `zeros(h,k)` will create a all-zero-matrix of size h -by- k
- `ones(h,k)` will create a all-one-matrix of size h -by- k
- `zeros(h,1)` will create a zero column vector of size h -by-1
- `ones(h,1)` will create a one column of size h -by-1
- `eye(n)` will create a n -by- n identity matrix
- the code `[u v]` will stack two column vectors \mathbf{u}, \mathbf{v} horizontally to form a matrix $[\mathbf{u} \ \mathbf{v}]$
- the code `[u; v]` will stack two column vectors \mathbf{u}, \mathbf{v} vertically to form a vector $[\mathbf{u}^\top \ \mathbf{v}^\top]^\top$

END.