Update 2022-July-1 (new measure to reduce workload)
under the new grading formula, for A7 and A9, you only need to pick one
That is, the assignment part of the grade will be A1 + A2 + ... + A6 + A8 + max{ A7, A9 }

CO327 (2022Spring) Assignment 7 - Image completion

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- Assignment deadline: July-10 23:55.
- Submit your electronic copy (a single PDF and MATLAB code) to the dropbox in Waterloo LEARN.
- Your MATLAB code has to be executable. That is, error free. Points will be deducted for code with error(s).

1 Image completion (100 points)

In this assignment, you will perform an image processing operation called "image completion".

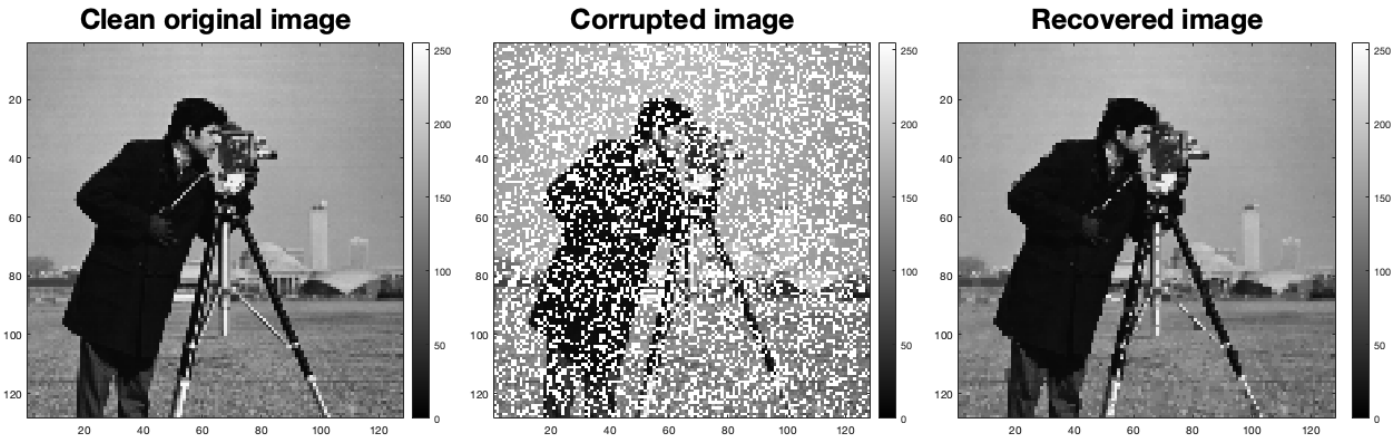

Figure 1: The image completion: you are given a corrupted image, and your goal is to repair it so that the recovered image looks like the original clean image.

1.1 About the data

You are given a "corrupted image" $X_{\text{corrupted}} \in \mathbb{R}^{n\times n}$. In the dataset, there are two images, the "cammerman" (a 128-by-128 matrix, as shown in Figure 1) and the "mario" (a 50-by-50 matrix, as shown in Figure 2). Your goal is to repair such image. That is, you want to get $X \in \mathbb{R}^{n\times n}$ that represent the repaired $X_{\text{corrupted}}$.

Figure 2 shows the corrupted mario image contains numbers from 0 to 254, and 255 (white color) represents a broken pixel value.

Refer to the lecture on image inpainting, the mathematics of such "completion" is to solve the following optimization problem

$$\begin{array}{ll} \min & \|\mathbf{E}\mathbf{x}\|_1 \\ \text{s.t.} & \mathbf{S}\mathbf{x} = \hat{\mathbf{x}}_\Omega \\ & \mathbf{x} \geq \mathbf{0} \end{array} \qquad (*)$$

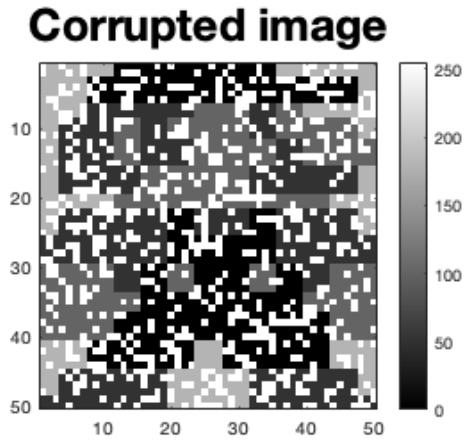

Figure 2: The corrupted mario image in the data.

where

- $\mathbf{X} \in \mathbb{R}^{n \times n}$ is the repaired image, which is a matrix
- $\mathbf{x} \in \mathbb{R}^{n^2}$ is the vectorized image. It is the vectorized version of $\mathbf{X}$. Here, we perform vectorization to turn the matrix $\mathbf{X}$ to a vector so that we can run LP code to find it.
- $\|\mathbf{E}\mathbf{x}\|_1$ is called the "Total Variation" (TV) of the vector $\mathbf{x}$, where $\mathbf{E}$ is a given matrix that compute the "variation" of the vector $\mathbf{x}$.
- $\mathbf{S} \in \mathbb{R}_+^{|\Omega| \times n^2}$ is a sub-matrix of $\mathbf{I}_{n^2}$ consists of rows labeled in the set Ω, which label which pixel we have observed in the image
- $\hat{\mathbf{x}}_\Omega \in \mathbb{R}_+^{|\Omega|}$ is the clean part of the observed image, with $|\Omega| < n^2$ number of entries.

1.2 Your task (50 points)

Given $\mathbf{E} \in \mathbb{R}^{2n(n-1) \times n^2}, \mathbf{S} \in \mathbb{R}_+^{|\Omega| \times n^2}$, and $\hat{\mathbf{x}}_\Omega \in \mathbb{R}_+^{|\Omega|}$, write a program to solve Problem $*$. Plot the recovered image. There are 2 images in the data file, try with `mario.m` (the smaller one) first. See `a7_main.m` in the data file for details.

2 Bonus part (50 points)

Solve the following

$$\min \|\mathbf{E}\mathbf{x}\|_p + \lambda \|\mathbf{S}\mathbf{x} - \hat{\mathbf{x}}_\Omega\|_q \text{ s.t. } \mathbf{x} \geq \mathbf{0} \qquad (**)$$

for $p, q \in \{1, \infty\}$, $\lambda \geq 0$ is called regularization parameter. Write a series of programs to solve such problem for different p, q, plot these recovered images, compare the recovered image to the one from solving Problem $(*)$.

General hint If you implemented everything correctly, the recovered image should looks like the original clean image.

END of assignment.