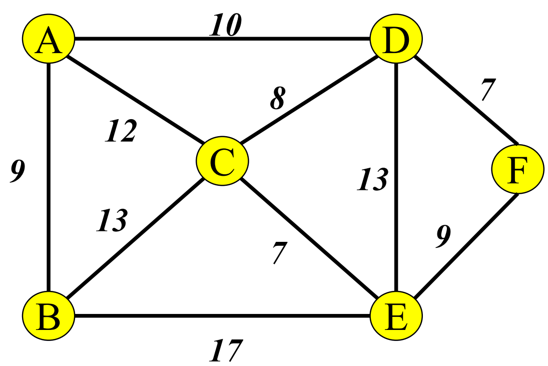
Update 2022-July-1 (new measure to reduce workload)
under the new grading formula, for A7 and A9, you only need to pick one
That is, the assignment part of the grade will be A1 + A2 + ... + A6 + A8 + max{ A7, A9 }

CO327 (2022Spring) Assignment 9 - Location planning

Lecturer: Andersen Ang

May 29, 2022

- Assignment deadline: July-24 23:55.
- Submit your electronic copy (a single PDF) to the dropbox in Waterloo LEARN.



1 Set cover problem (5 points)

Introduction Given a graph, you want to decide where to place hospitals.

- A graph $G(V,E)$ consists of two sets: the node set V and the edge set E.
- There are six nodes $V = \{A,B,C,D,E,F\}$ (or equivalently $V = \{1,2,3,4,5,6\}$). Their locations are the potential sites to build a hospital. You can treat these nodes as "city" in a region.
- Let $d(A,D)$ denotes the *minimum* distance between cities A and D.
  - In this question, the edges represent the physical distance between two particular nodes *that are directly connected*. For example, $d(A,D) = 10$ physically means that there is a 10-km road constructed between cities A and D.
  - The distance from a node to itself is defined as 0. That is, $d(A,A) = d(B,B) = \cdots = d(F,F) = 0$.
  - For the distance between cities that are not directly connected by an edge, we consider the **shortest distance** between them. For example, if one travels from F to A, the person will travel from F to A by passing D, and the total distance between F to A is thus $10 + 7 = 17$. Here $d(A,F) = d(F,A) = 17$ and it is the smallest distance according to the graph.

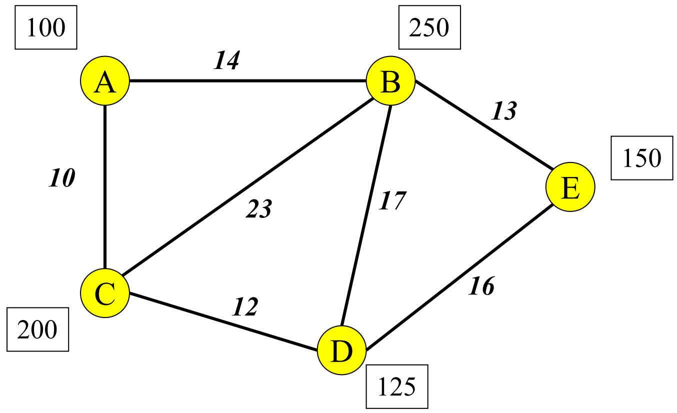
The problem Now we want to decide where to place hospitals under a constraint: we want to make sure that the travel distance between any cities to **its closest hospital** is under a upper bound b. Our goal here is to decide where to place the hospital such that the number of hospital is minimized. Note that when modeling the distance between two cities, we consider the **shortest distance**.

Examples For $b = 15$, we need 1 hospital, we build hospital in C. For $b = 10$, there is no solution. For $b = 9$, we need 3 hospitals, we build hospital in A, D, F.

Your tasks Write down such problem as a an optimization problem for $b = 10$, then solve the problem.

2 p-median problem (6 points)

Now consider a graph of a new region with five cities $\{A, B, C, D, E\}$.



We now decide where to build hospital based on p-median model.

- Here the weights on the edges $w_{i,j} \geq 0$ represents the cost of travel between vertices i and j. For example, the travel cost between A and B is 14.
- We introduce positive weights to each nodes d_i for all i that represents the amount of demand at node i. For example, the demand of A is 100.
- We assume all hospitals have no capacity limit (it can serves infinite many patients), so when you assign a node to a facility, you can assign all the demand of that node to the same facility.

You wish to place $p = 1$ facilities to minimize the total travel cost between a demand node and the location in which a facility was placed. Formulate this 1-median problem and solve it.

Hints

- As we assumed all hospitals have no capacity limit, we can let $z_{ij} \in \{0, 1\}$ presents allocating patient i to facility j. Then we let $y_j = 1$ if facility located at j and $= 0$ otherwise.
- Refer to the lecture notes on p-median problem, there are two subsets in the node set V as $V = \{I, J\}$ where I denotes the vertex set of demand nodes and J denotes the vertex set of potential facility location. In this problem, $I = J = V = \{A, B, C, D, E\} = \{1, 2, 3, 4, 5\}$.
- You can choose to use symbol w_{ij}, d_i instead of the explicit numeric values in the graph to write down the optimization problem. For example, suppose you want to write down

$$100y_1 + 250y_2 + 200y_3 + 125y_4 + 150y_5,$$

which is the sum of demand of each node multiplied by y_j, you can write it as compactly as

$$\sum d_j y_j.$$

For Q3 and Q4, we consider a general graph $G(V, E)$.

3 Capacitated p-median problem (7 points)

Instead of assuming hospitals have no capacity limit in the p-median problem, we consider a more realistic scenario: let $C > 0$ be the maximum capacity (number of patients) of each hospital is able to serve. Model the hospital planning problem again with such newly introduced capacity of constraint.

Hints Let x_{ij}: amount of demand at demand node i serviced by a facility placed at j, and then let $y_j = 1$ if facility located at j and $= 0$ otherwise.

4 Capacitated p-center problem (11 points)

Now, you want to place p facilities to minimize the **maximum distance** between any demand node and its servicing facility, under the same constraints (capacity of the facilities.) Formulate this problem (known as the capacitated p-center problem).

Hints Consider four variables z, x, y, t

- x_{ij}: amount of demand at demand node i serviced by a facility placed at j.
- $y_j = 1$ if facility located at j and $= 0$ otherwise .
- $w_{ij} = 1$ if demand node i is assigned to facility located at j and $= 0$ otherwise.
- t: the max distance between any demand node and its servicing facilities.

END of assignment.