CO327 (2022Spring) Midterm Assignment

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Instruction

- Submit your solution in **one single PDF**, write down your name and student ID.
- LaTeX is not necessary.
- Submit your PDF to the dropbox in Waterloo LEARN before the **deadline** June-29 23:55 (EDT).
  - **Late submission get zero point**.
  - If you re-submit, only the newest submission will be considered and your previous submission(s) will be ignored.
- Use of MATLAB is allowed, **you DO NOT need to submit the MATLAB code**.

1 Coins Change Problem (7 points)

Problem description A country uses N coins with denominations $\{a_1, a_2, ..., a_N\}$. Given a value V, find the minimum number of coins that add up to V.

Example The coins in Japanese Yen are $\{1, 5, 10, 50, 100, 500\}$. Peter wants $V = 678$. The minimum number of coins required to make V is 9:

One 500 yen coin	Two 10 yen coin
One 100 yen coin	One 5 yen coin
One 50 yen coin	Three 1 yen coin

Your tasks

- Formulate the problem (NOT the example) as a linear program/integer program. State clearly your decision variable(s), objective function and constraint(s).
- Solve the problem for $N = 3$ with $a_1 = 0.5, a_2 = 1, a_3 = 2$ and $V = 5.5$.

2 Employee scheduling (13 points)

You have a 24-hour supermarket. The supermarket has the following minimal requirements for cashiers:

Period	1	2	3	4	5	6
Time of the day (in 24-hour format)	3-7	7-11	11-15	15-19	19-23	23-3
Wages	110	100	100	107	107	110
Minimum no. of cashiers needed	2	10	4	12	4	2

Table 1: The wages and minimum number of cashiers for each work period.

Period 1 follows immediately after period 6. A cashier works 8 consecutive hours, starting at the beginning of one of the six periods. Different periods have different wages.

1. Your goal is to determine a daily employee worksheet (how many cashier in each time period) which satisfies the requirements with the least wages. Formulate this problem as a linear program/integer program. State clearly your decision variable(s), objective function and constraint(s). (8 points)

2. Solve the problem, how many cashier you need in each period? What is the total wages? (2 points)

3. At the solution, identify which constraint is active and which constraint is inactive. (3 points)

3 Linear Programming (22 points)

Consider the following problem

$$\begin{array}{ll} \min & -2x_1 - x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 3 \\ & \left|x_1 + 4x_2\right| \leq 5 \\ & x_1 \geq 0 \\ & -x_2 \leq 0 \\ & |x_3| \geq 0 \end{array} \tag{1}$$

1. Convert (1) to canonical form

$$\begin{array}{ll} \max\limits_{\mathbf{x}} & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{array} \tag{1'}$$

2. Identify any redundant constraint in (1′).
3. Solve (1′) , give an optimal solution point $\mathbf{x}^*$.
4. State the active set S of $\mathbf{x}^*$. Based on $\mathbf{A}_S\mathbf{x}^* = \mathbf{b}_S$, what can you tell about the uniqueness of the solution to (1′)?
5. Convert (1′) to the following form

$$\begin{array}{ll} \max\limits_{\mathbf{x}} & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array} \tag{1''}$$

   Then derive the symmetric dual of (1″) and solve it. Verify the duality and the complementary slackness between the primal and the dual problems.

Course survey

- What do think about the difficulty of the course? Pick one:

  Too easy Slightly easy OK Slightly hard Too hard

- What do think about the amount of assignment? **(Remember, there is no sit-in midterm and sit-in final exam)** Pick one:

  Too few Few OK Many Too many

- What do think about the teaching? Pick one:

  Too bad Bad OK Good Too good

- Write down one thing you like about the course.
- Write down one thing you don't like about the course.
- Write down any other comment(s).

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